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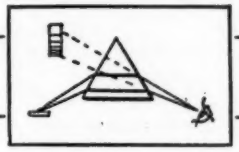

SEPTEMBER
1934

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Journal of the
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VOLUME 24 SEPTEMBER, 1934 NUMBER 9

The Appearance of Atoms as Determined by X-Ray Scattering

E. O. WOLLAN AND A. H. COMPTON, *University of Chicago*
(Received July 23, 1934)

Photographs are shown which are the images of atoms of helium, neon and argon as obtained by x-ray diffraction, magnified about 2×10^8 times. The images are obtained by photographing a rotating template whose shape is calculated by a mathematical transformation of our measured values of the x-rays scattered by the respective gases. This mathematical-mechanical procedure corresponds to the lens which forms the image when a microscope is used. The images formed by our procedure should be true representations of the electron distributions in the atom, except for the limited resolving power and certain minor aberrations. The photographs show the helium atom as a diffusely continuous region filled with electricity. In neon, the inner group of K electrons is clearly distinguishable from the L electron group. The resolving power is insufficient to distinguish the K and L groups of electrons in argon, but does separate these from the M electrons. The appearance of these atoms is in good accord with modern quantum theory of atomic structure.

THE study of the diffraction of x-rays by crystals and gases has made it possible to obtain a great deal of information regarding the arrangement and spacing of the atoms in gas molecules and in complicated crystals. It has also given us what is probably the most direct method of obtaining the distribution of the electrons within an atom. In the present paper it will be shown how one can obtain, with the aid of x-rays, what, in effect, corresponds to a photograph of an atom. It is, of course, evident that one cannot photograph an atom by the straightforward method of projecting its image on a photographic plate by means of lenses, since there are no lenses which will focus x-rays. The details of the appearance of atoms are obtained from x-ray diffraction data and the photograph is produced by a combined mathematical and mechanical procedure.

Many examples could be cited in which the details of the appearance of an object cannot be obtained by direct photography, but a study of its diffraction pattern can be made to give the desired results. Our knowledge of star diameters, for example, is based on such information. A simple illustration of the way a photograph of an object can be derived from its diffraction pattern is given by Michelson.[1] By means of a calculation based on the diffraction pattern produced by a single slit, one can construct a new diffraction screen, and if parallel light is passed through this, an image of the original slit is obtained. W. L. Bragg[2] has used a method

[1] A. A. Michelson, *Studies in Optics* (University of Chicago Press, 1928), p. 60.

[2] W. L. Bragg, Zeits. f. Krist. **70**, 489 (1929); *The Crystalline State* (Macmillan, 1934), p. 227.

The photographs obtained by Bragg do not show as great detail in the structure of the atoms as those here presented. This is doubtless due in part to the fact that the photographic method of obtaining the image which he uses must result in considerable loss of detail; but there is also the essential limitation of the crystal method that

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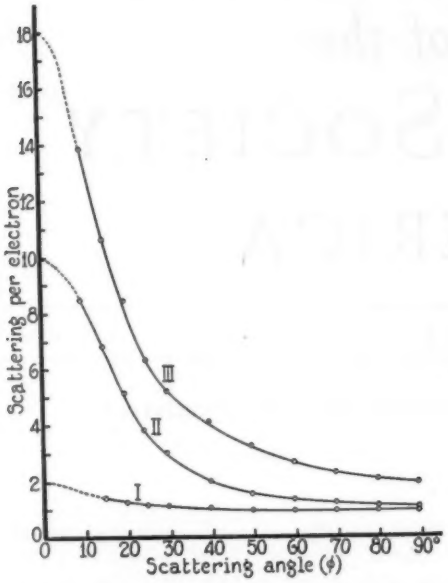

FIG. 1. Intensity of x-rays scattered by gases. I, helium; II, neon; III, argon.

almost identical in principle for securing a photographic image of the projection on a plane of the atoms in a crystal.

We have used a somewhat similar procedure in obtaining pictures of atoms. The atom is the diffraction screen for which we measure the intensity of the diffraction pattern. The photograph is a result of a mathematical and mechanical procedure which, in effect, does the work which the lens does in a camera.

In the experimental set-up a parallel beam of x-rays falls on an atom which gives its characteristic diffraction pattern or in other words an angular distribution of intensity of scattering. We cannot, however, isolate a single atom, nor could we measure the very feeble scattering from it. Actually, we observe the x-ray diffraction from a gas consisting of n atoms distributed at random, which diffraction pattern we know is the same as that due to one atom except that it is n times as intense. The photograph which we obtain then represents an average picture of these n atoms.

Fig. 1 shows the diffraction data obtained[3] by scattering x-rays of wave-length $\lambda=0.71$A from helium, neon and argon. The quantity S represents the contribution to the total scattered intensity made by each electron measured in units of the Thomson scattering by a single free electron. From these data one can calculate the distribution of the diffracting medium (the electrons) within the atom as a function of its radius. If we represent the number of electrons which lie at distances between a and $a+da$ from the center of the atom by $U(a)$ we have previously shown[4] that it is related to the experimental quantity S by the Fourier integral,

$$U(a)=\frac{2Za}{\pi}\int_0^\infty\left\{\frac{S-R}{Z-R}\right\}k\sin(ak)dk, \quad (1)$$

where Z is the atomic number, $S=I_\varphi/ZI_e$, I_e being the Thomson scattering by a single free electrons, $k=(4\pi/\lambda)\sin(\varphi/2)$, and $R=(1+(h/mc\lambda)\text{ vers }\varphi)^{-3}$. Using the data for S given in Fig. 1, $U(a)$ can be calculated from Eq. (1) by means of a graphical integration. The result of

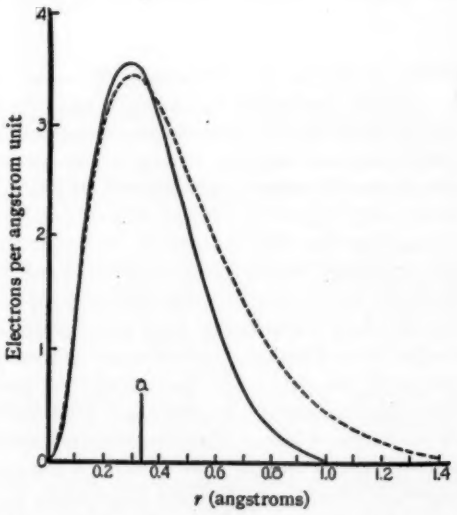

FIG. 2. Radial electron distribution in helium. Solid line from experiment, broken line from wave mechanics according to Hartree.

the atoms which are photographed are in thermal motion about their neutral positions. This, of course, makes a blurred image. Our method, being based on diffraction by gaseous molecules, assumes a random arrangement, and is hence unaffected by the thermal motion. Thus our practical resolving power almost equals the theoretical value.

[3] E. O. Wollan, Phys. Rev. **37**, 862 (1931).

[4] A. H. Compton, Phys. Rev. **35**, 925 (1930).

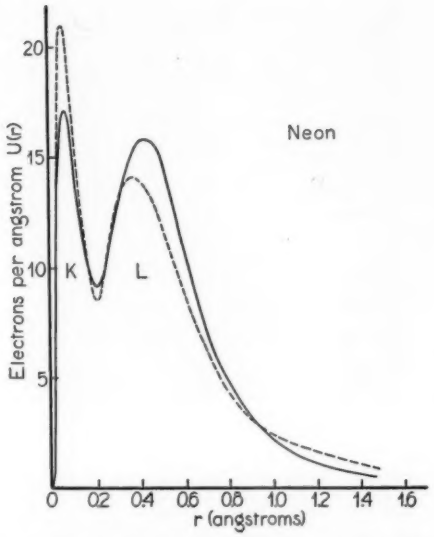

FIG. 3. Radial electron distribution in neon. Solid line from experiment, broken line from wave mechanics according to Hartree.

such a calculation for helium is shown by the solid line in Fig. 2. The dotted line is a similar curve calculated on the basis of wave mechanics and one sees that the electron distribution calculated in this way is in good accord with the experimental curve. On the basis of the older Bohr theory all the electrons in the atom would lie at a distance $a=0.33$A from the center of the atom, which does not accord with the experimental results.

Fig. 3 shows the electron distribution curve for a neon atom calculated in a similar way from Eq. (1) and the data of Fig. 1. An especially interesting feature of this curve is the fact that the K and L electrons are resolved. A similar analysis of argon as shown in Fig. 4 fails to separate the K and L groups, the approximate position of which can be seen from the dotted curve obtained from wave mechanics. This is, of course, due to the fact that these details of the atom become smaller and more difficult to resolve as the atom becomes heavier.[5]

It is interesting in this connection to make a calculation of the resolving power of our apparatus. To do this, let us assume that two electrons can be distinguished if the waves scattered by them traverse path differences of more than half a wave-length. At the most favorable orientation of the two electrons, this will occur when

$$\lambda/2=2\delta \sin(\varphi/2)$$

or

$$\delta=\lambda/4 \sin(\varphi/2),$$

where δ is the minimum distinguishable distance. Taking the value of λ and the maximum value of φ for which the data of Fig. 1 were obtained, we get $\delta=0.25$A. This value of δ shows why the K and L electrons were resolved in the neon atom and not in the argon atom.

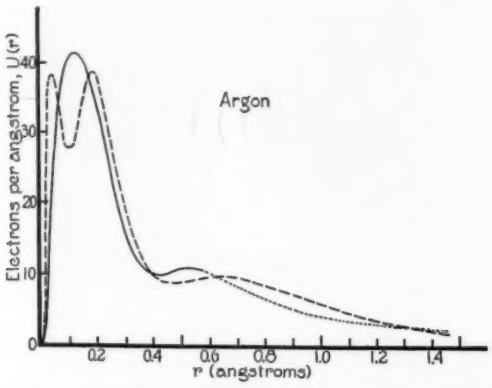

FIG. 4. Radial electron distribution in argon. Solid line from experiment, broken line from wave mechanics according to Hartree.

Although the curves of Figs. 2, 3 and 4 represent, in one way, our knowledge of these atoms, it is not easy on the basis of them to construct in one's imagination a picture of the atoms. An image of an atom as it would appear through an x-ray microscope would correspond to the projection of the electron atmosphere on a plane perpendicular to the direction of observation. To do this, let us represent the atom in cylindrical

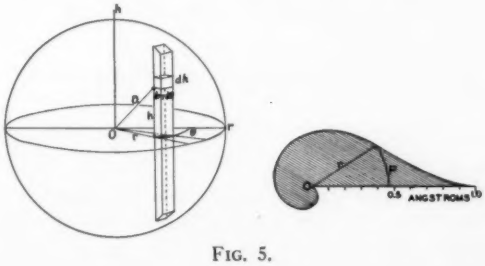

FIG. 5.

[5] For a more detailed account of these results, cf. E. O. Wollan, Rev. Mod. Phys. **4**, 243 (1932).

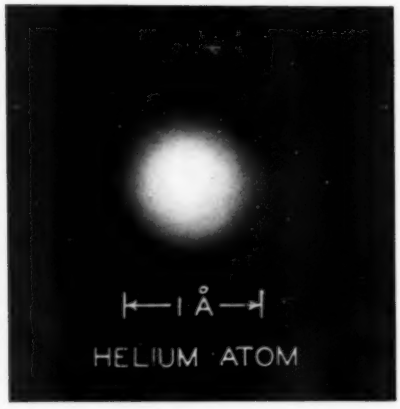

FIG. 6.

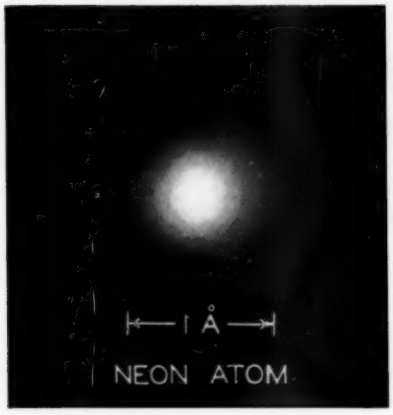

FIG. 7.

FIG. 8.

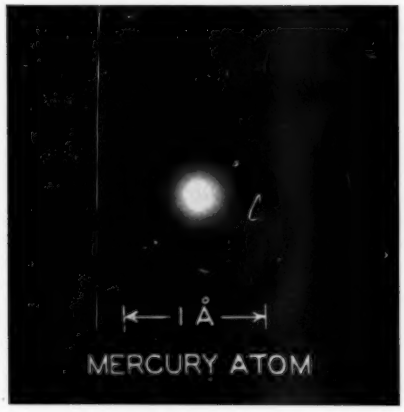

FIG. 9.

FIGS. 6, 7 AND 8 show "appearance" of atoms of helium, neon and argon, respectively, as observed with x-rays of wave-length 0.71A and a spectrometer working to an angle of 90°.

FIG. 9 shows "appearance" of mercury atom, based on Thomas-Fermi electron distribution, as supported by x-ray scattering experiments. This picture represents closely, except for differences in scale, the "appearance" of any atom heavier than argon.

coordinates as in Fig. 5, and let us project the electron density on the plane $h=0$. The number of electrons projected on the element of area $r\cdot d\theta\cdot dr$ will then be

$$P(r)\cdot r\cdot d\theta\cdot dr=2\int_0^\infty \rho\cdot r\cdot d\theta\cdot dr\cdot dh, \qquad (2)$$

where $\rho=U(a)/4\pi a^2$ is the volume density of distribution of the electrons and $P(r)$ represents the number of electrons per square Angstrom in the plane of projection. Putting $h=(a^2-r^2)^{\frac{1}{2}}$, Eq. (2) becomes

$$P(r)=\frac{1}{2\pi}\int_r^\infty \frac{U(a)}{a(a^2-r^2)^{\frac{1}{2}}}da. \qquad (3)$$

Values of $P(r)$ for a given atom can be determined from Eq. (3) by using the values of $U(a)$ given in curves of the form shown in Figs. 2, 3 and 4. On the other hand, $P(r)$ can be represented directly in terms of the experimentally observed quantity S by combining Eqs. (1) and (3). This gives

$$P(r)=\frac{Z}{\pi^2}\int_r^\infty\int_0^\infty \left\{\frac{S-R}{Z-R}\right\}^{\frac{1}{2}} \frac{k\sin(ak)}{(a^2-r^2)^{\frac{1}{2}}}dkda, \qquad (4)$$

where one sees that the right-hand side of this equation depends only on the experimentally observed intensity S and other known or measurable quantities, and not on any theory of atomic structure.

From the Fourier integrals represented by Eq. (3) or (4), one can calculate $P(r)$ as a function of r by graphical integration. Making such a calculation on the basis of the experimental values of S in Fig. 1, we proceed to represent the function photographically. A simple method of doing this is the following: We plot $P(r)$ in polar coordinates against r, as shown by the shaded figure in Fig. 5 for the case of helium. Since $P(r)$ represents the number of electrons per square Angstrom, the arc shown in the figure will correspond to the total number of electrons at a distance r from the center which is also proportional to the scattering power of the atom at this value of r. Hence, a template of this form cut from a piece of white paper will scatter light in the same proportion that the corresponding atom scattered x-rays. If this template is placed on a black background and rotated about its center, a photograph of it will, in effect, be a photograph of the corresponding atom.

Figs. 6, 7 and 8 show the photographs which have been obtained according to this procedure for the atoms of helium, neon and argon. These photographs represent the atoms as observed with x-rays of wave-length 0.71 Angstrom, in which, instead of a lens, the image has been formed, as just described, by means of a mathematical and mechanical method. The magnification, which is equal to the ratio of the unit on which r is plotted in Fig. 5, to the unit in which the atom is measured, is in the present instance, equal to about 2×10^8 times. The resolving power of the apparatus as we have noted above was sufficient to separate the K and L electrons in neon, but in argon, due to its greater compactness, these are not resolved. The K electrons in neon are shown in the photograph by the very bright center, and the L electrons are represented by the diffuse outer ring. In argon the bright center corresponds to the combined K and L electrons, and the outer diffuse part to the M electrons.

Fig. 9 shows a similar picture for an atom of mercury.[6] The fact that the atom appears so concentrated is to be expected for so heavy an atom. This fact is however overemphasized in the picture, due to the inadequacy of the photographic process to represent large variations of density. For mercury the rays scattered from the center of the atom are so very intense that it becomes impossible to represent photographically the very much less scattering from the outer part of the atom. This shortcoming of the photographic process applies also to the case of the lighter atoms but to a very much less extent. It is a difficulty with which any photographer must contend.

There are other inaccuracies which may be introduced into our photographs due to certain approximations in our mathematical-mechanical method. These inaccuracies can hence be considered as analogous to the aberrations obtained with a lens. The photographs shown in Figs. 6–9 are free from these aberrations if one can assume (1) that the atoms are spherical and (2) that every electron has the same chance of occurring at any place in the atom. Auxiliary evidence indicates that assumption (1) is valid for all atoms of the noble gas group, and that (2) is exact for helium, very approximately valid for argon, and not seriously in error for neon. Thus the appearance of the helium, neon, and argon atoms, as shown by these photographs, should correspond very closely to their true form.

The x-ray diffraction experiments under discussion thus supply us with just the same information that would be obtained if we could look at the atom with an x-ray microscope. The size of the atom is clearly observed, and, as we have seen, the resolving power of our apparatus is also sufficient to distinguish various groups of electrons within the atom.

Photographs representing the *theoretical* electron density (Schrödinger's function $\psi\psi^*$) for various states of hydrogen-like atoms have been published by White.[7] These bear a close resemblance to the photographs of the *experimentally observed* atoms pictured here. It is significant that our pictures are in good accord with predictions of the new quantum mechanics.

[6] The x-ray scattering data on which the mercury atom photograph is based, were obtained by P. Scherrer and A. Stäger, Helv. Phys. Acta **1**, 518 (1928).

[7] H. E. White, Phys. Rev. **37**, 1423 (1931).

Visual Spectrophotometry*†

K. S. GIBSON,‡ *National Bureau of Standards*

(Received May 24, 1934)

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I. INTRODUCTION

THE optical principles involved in visual spectrophotometers can be illustrated by describing in some detail certain of the instruments and equipment which are now manufactured or used in the measurement of spectral transmission or reflectance. It is not attempted to survey the field completely nor to give an historical account of the development of visual spectrophotometric methods. Such an effort was made in the O. S. A. spectrophotometry committee report in 1925.[1] However, the various methods described herein are believed to include most of the new or improved designs effected in visual spectrophotometers since that date.

The instruments selected for discussion illustrate most of the devices that have been used to secure the necessary two-part photometric field and the methods employed to vary the brightness of one or both parts of the field in a measur-

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‡ Acknowledgment is made for the use of the several cuts, the sources of which are mentioned in the text.

[1] K. S. Gibson, chairman, J. O. S. A. and R. S. I. **10**, 169–241 (1925). See also report of progress committee on radiometry and photometry, J. O. S. A. and R. S. I. **11**, 359–361 (1925).

able manner. These are essential conditions of all visual photometry[2] and arise from the well-known fact that the eye is totally incapable of any reliable judgment regarding absolute brightness but is able to decide with considerable precision whether two properly juxtaposed photometric fields of the same chromaticity are equally bright. Combining these essential photometric conditions with means for dispersing the light and isolating a narrow spectral region of the desired wave-length gives the fundamental basis of all visual spectrophotometers.

After the instruments and methods are described, the various factors will be considered which are of importance in the elimination of errors, such as proper methods of illumination, reference standards for reflectance measurements, wave-length calibration, elimination of stray light, slit-width corrections, and means of checking the reliability of the photometric scale and the over-all accuracy of the instrument.

II. Typical Instruments and Methods

All of the instruments described have photometric fields which will be classified as type I or type II, depending on their method of formation. These two types are illustrated in Fig. 1, which

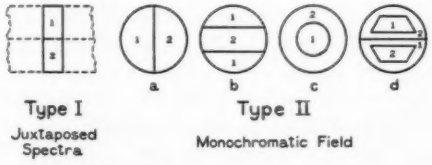

FIG. 1. Illustrating various forms of two-part photometric field used in spectrophotometry.

is taken from the spectrophotometry committee report to which reference has just been made.

Type I, known as the "juxtaposed spectra" type, is so designated because the photometric field consists of a selected portion of two juxtaposed spectra. These spectra are formed in the focal plane of the telescope lens of the spectrometer and are viewed by a magnifying eyepiece. The horizontal dividing line between the two spectra is produced by some optical device external to the spectrometer, either placed at the collimator slit or imaged upon that slit. The amount of spectrum viewed at one time may be varied by changing the width of the shutter indicated by the vertical lines in Fig. 1, type I. It is apparent that, in general, the color of both halves of the field will vary from left to right and that any sudden change in brightness with wave-length will be visible in the field.

In type II the dividing lines of the field are produced by some optical device either placed between the collimator and telescope lenses or imaged between them from some region outside of the spectrometer. The eye is placed at the exit slit of the instrument close to the focal plane of the telescope lens. In this position any object placed in the path of the beam between the collimator and telescope lenses or close to them will appear sharply defined. In particular a two-part photometric field may readily be formed by placing an appropriate optical device in this position or imaging such device in this location. Various means for doing so are illustrated in the several instruments. The photometric field thus obtained is called the "monochromatic field" type because, regardless of the slit-widths used, the whole field is of the same chromaticity. This method of securing the photometric field is also referred to as the "Maxwellian field of view."

1. Polarization Methods

Polarization methods have perhaps been more commonly used than any other as a means of varying the brightness of the photometric field. Three different types of polarization instruments are illustrated: (1) The König-Martens spectrophotometer made by Schmidt and Haensch, with auxiliary equipment designed at the National Bureau of Standards, (2) the Martens photometer combined with spectrometer and auxiliary equipment, made by (a) the Bausch and Lomb Optical Company and (b) the Gaertner Scientific Corporation, and (3) the spectrophotometric equipment designed by Dowell and made by Adam Hilger, Ltd.

[2] Excluding consideration of the well-known "flicker" method of heterochromatic photometry and other methods which have been rarely used in spectrophotometry.

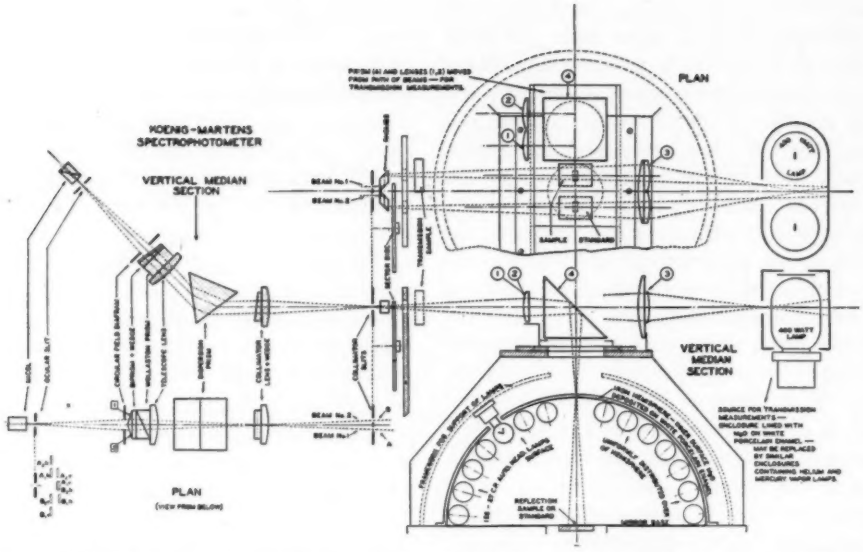

FIG. 2. König-Martens spectrophotometer, with auxiliary equipment designed at the National Bureau of Standards.

(1) König-Martens spectrophotometer,[3] with auxiliary equipment designed at the National Bureau of Standards

This apparatus is outlined in Fig. 2, which is a reproduction of Fig. 3 of Bureau of Standards Research Paper No. 30.[4] Light entering the collimator slits, A and B, forms beams 1 and 2. These beams follow the usual course through the collimator lens, dispersing prism and telescope objective. Cemented to the latter lens is a combination of wollaston prism, wedge[5] and biprism. The combined action of the wollaston prism and biprism is to form four spectral images of each of slits A and B in the plane of the ocular slit. The angle of the biprism is chosen, with regard to the separation of slits A and B and the construction of the wollaston prism, so that two of the eight images overlap at the ocular slit. The designations, A_1v and B_2h, indicate that one of these images is formed by light entering slit A and passing through part 1 of the biprism. It is plane-polarized with electrical vibrations in a vertical plane. The other image is formed by light from slit B passing face 2 of the biprism and is polarized with vibrations in a horizontal plane.

Looking through the ocular slit one sees the surface of the biprism uniformly illuminated by a mixture of light of wave-length range determined by the widths of the collimator and ocular slits, the mean wave-length corresponding to the position of the ocular slit in the spectrum. The biprism edge forms a vertical dividing line in this photometric field, which is of type II-*a*, and the lights in the two halves of the field are plane-polarized in directions mutually perpendicular to each other. By turning the nicol between the eye and the ocular slit, the two parts of the field may be matched in brightness.

For spectral transmission measurements three different illuminants are used, *viz.*, the mercury arc in quartz, the helium lamp and the incandescent lamp. Only the latter is shown in the diagram. Each of the three illuminants is

[3] F. F. Martens and F. Grünbaum, *Über eine Neukonstruktion des Königschen Spektralphotometers*, Ann. d. Physik **12**, 984–1003 (1903). See also Schmidt and Haensch, Catalog II, Spektralphotometer, pp. 13–21, October, 1930.

[4] H. J. McNicholas, *Equipment for Routine Spectral Transmission and Reflection Measurements*, Bur. Standards J. Research **1**, 793–857 (1928). See also K. S. Gibson, *Spectrophotometry at the Bureau of Standards*, J. Opt. Soc. Am. **21**, 564–587 (1931).

[5] The purpose of the wedges in the collimator and telescope is to prevent passage to the eye of certain multiply reflected rays from the optical surfaces.

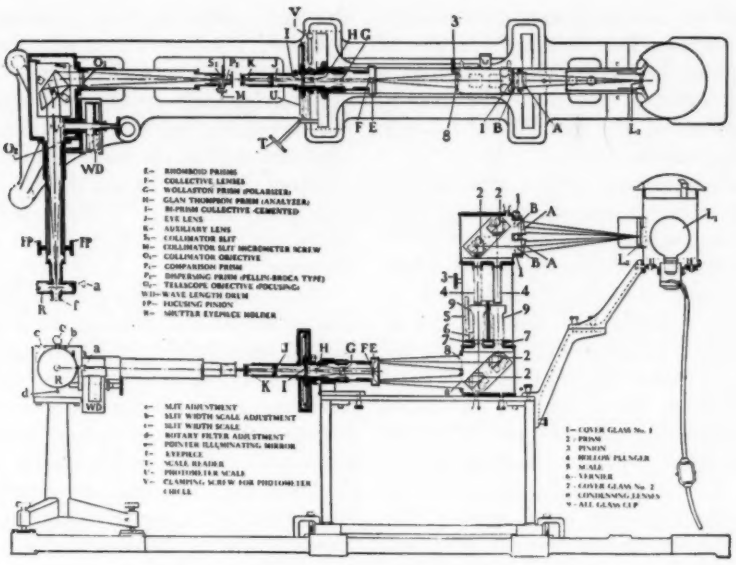

FIG. 3. Bausch and Lomb spectrophotometric equipment.

mounted in a small enclosure, the inside surface of which is coated with MgO. In each case the light used for the transmission measurements is taken from the diffusing rear surface of the enclosure; this is collimated by the combination lenses (3) and enters the collimator slits, A and B, as shown.

The specimen whose transmission is to be measured is placed in beam 1 as shown and the angle of match, θ_1, of the nicol is read; the specimen is then transferred to beam 2 and the angle of match, θ_2, determined, after which the sample is returned to beam 1 and the determination of θ_1 repeated. The transmission, T, of the sample is given by the relation

$$T = \cot \theta_1 \tan \theta_2. \qquad (1)$$

The transmission may also be measured by taking readings first with the sample placed in either beam and then with it removed from both beams. If θ_0 is the angle of match in this latter case, T is given by the relations

$$T = \tan^2 \theta_0 \cot^2 \theta_1 \qquad (2)$$

$$= \cot^2 \theta_0 \tan^2 \theta_2. \qquad (3)$$

The interchange method of measurement, Eq. (1), is superior to the simple substitution method, Eqs. (2), and (3), because errors due to partial polarization of light by the sample are thereby largely eliminated, as noted below.

The mercury and helium illuminants afford ready means of checking the wave-length calibration of the spectrophotometer; they also enable measurements of transmission to be made at certain wave-lengths free from slit-width or wave-length error. These wave-lengths are: Hg 578 (that is, 576.9 and 579.0), 546.1, 435.8, 404.7 mμ and He 667.8, 587.6, 501.6 and 447.1 mμ.

The rotating sector, shown in the diagram between the collimator slit and the transmission sample, serves two purposes: (1) It enables a direct check to be made on the reliability of the photometric scale of the instrument. A number of sectors were prepared having respective apertures such as to give transmissions of approximately 0.01 to 0.80. These apertures were accurately determined mechanically. Any desired sector may be placed in position, rotated rapidly enough to eliminate flicker and its transmission determined photometrically in the same manner as for the usual transmission sample.

(2) To measure low transmissions, the 0.10 or 0.01 sector is placed in the blank beam and the

transmission of the sample is measured relative to that of the sector. This brings the angles of match away from the extinction points into a more suitable region of the scale and greatly extends the range of the instrument for low transmission measurements.

For spectral reflectance measurements, the sample and reference white standard[6] are placed, as shown, at the center of the base of a hemisphere whose interior surface is coated white with MgO and studded with 156 small lamps so that in effect the sample and standard are in completely diffused illumination. The light reflected at right angles from the sample and standard, respectively, forms beams 1 and 2 and enters the spectrophotometer via the right-angled prism (4) and lenses (1) and (2). Sample and standard may be reversed in position by the observer and the apparent reflectance of the sample, R_s, relative to that of the standard, R_0, is given by the relation, analogous to Eq. (1),

$$R_s/R_0 = \cot \theta_1 \tan \theta_2.$$

The effect of polarization on the values of T or R_s/R_0 obtained with this equipment, is discussed by McNicholas. If X represents either of these quantities, the complete expression is

$$abX = \cot \theta_1 \tan \theta_2,$$

where a and b represent the respective changes in the state of polarization of beams 1 and 2, due to the introduction of the sample into the beam. It is shown that any initial differences in the beam intensities or any degree of polarization of the light emitted by the source[6a] along beams 1 and 2 does not affect the measurement of X; it is only the changes in polarization that may cause error. This error is usually negligible for the following reasons: (1) With this equipment the vast majority of samples do not affect the state of polarization of the light beams in either transmission or reflectance measurements. (2) In case the light reflected from a sample is partially polarized, because of structural regularities such as those exhibited by silk and velvet, the method of interchanging sample and standard greatly minimizes the resulting error. (3) If the sample is placed so that the plane of polarization of the reflected light as it enters the spectrophotometer is inclined at 45° to the principal planes of the instrument, the small residual error is completely eliminated.

The König-Martens spectrophotometer may also be used for measuring the spectral distribution of one illuminant relative to the other. A means of doing this, even for a fluctuating illuminant such as the carbon arc, has been described by Priest.[7] Various other methods of illumination, designed for special purposes, particularly for transmission measurements, are illustrated in catalogues issued by Schmidt and Haensch[8] and by Akatos, Inc.[9]

(2) Martens photometer and spectrometer combinations

(*a*) *Bausch and Lomb spectrophotometric equipment.* The Bausch and Lomb equipment consists of three units, (1) a spectrometer, (2) a Martens photometer and (3) auxiliary attachments for holding the sample and furnishing the proper illumination. One arrangement of apparatus is shown in Fig. 3, which is a reproduction of Fig. III-*b* from the Bausch and Lomb catalogue.[10] This arrangement facilitates the measurement of the transmittancy of liquids in varying thickness. The solution and solvent are placed in the cups (9). By raising the cups about the plungers (4) the thicknesses may be varied from less than 1 mm to 65 mm.

The illuminant consists of light from the incandescent lamp, L_1, diffused by the double ground-glass window, L_2. The beams are collimated by the wedges, A, and lenses, B, and follow the paths indicated through the solution

[6] The choice of reference standard is considered later.

[6a] It was found by McNicholas that the initial polarization of the light from the tungsten-incandescent, mercury-vapor, or helium illuminants (i.e., from the MgO diffusing surfaces where beams 1 and 2 originate) is less than 1 percent. Under the symmetrical illumination from the hemispherical source there is no appreciable polarization of the light reflected at right angles from a surface having no structural regularities.

[7] I. G. Priest et al., *Color and Spectral Composition of Certain High-Intensity Searchlight Arcs*, Bur. Standards Tech. Paper No. 168, 1920.

[8] Catalog II, pp. 13–21, October, 1930.

[9] Descriptive circular entitled, *Spectrophotometers according to Koenig-Martens and Koenig-Bechstein.*

[10] Advertising booklet entitled *Microscopes and Other Scientific Instruments*, pp. 232–245, 1929.

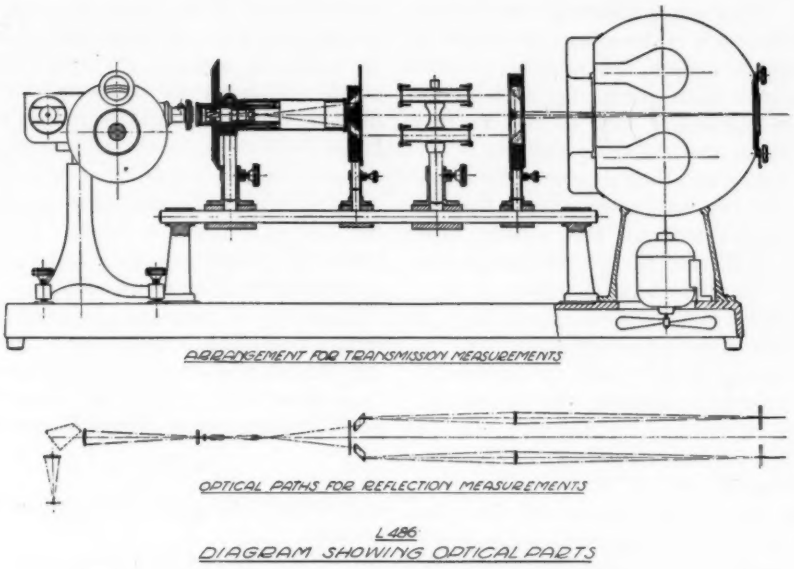

FIG. 4. Gaertner polarizing spectrophotometer.

and solvent, arriving at the respective entrances to the Martens photometer.

The action of the Martens photometer is similar to that already described for the König-Martens spectrophotometer. The wollaston prism, G, transmits two beams to the nicol, H, one from each entrance aperture, the beams being polarized in mutually perpendicular planes.[11] The intensities of the two beams may then be equalized by rotation of the nicol. The biprism, I, directs these two beams along the axis of the photometer and an auxiliary lens, K, focuses an image of the horizontal biprism edge onto the vertical collimator slit, S_1.

The spectrometer forms a spectral image of the collimator slit in the focal plane of the telescope objective. Viewing this image through the eyepiece, f, the observer sees a pair of juxtaposed spectra, the widths of which are controlled by a V-slide at the collimator slit. The length of spectrum viewed is regulated by a shutter in the eyepiece holder. The photometric field is thus of type I. The two halves of this field may be equalized in brightness by rotation of the nicol prism in the Martens photometer.

The transmission of a solution relative to that of the solvent may be determined at any wavelength by means of the same formulas as are used with the König-Martens spectrophotometer. The scale of the Martens photometer is calibrated not only in degrees but also in terms of T and $-\log_{10} T$, so that if θ_0 can be made exactly equal to 45° the instrument is direct reading. Other arrangements of apparatus for both transmission and reflectance measurements are illustrated in the Bausch and Lomb catalogue, Figs. I-b and II-b.

(*b*) *Gaertner polarizing spectrophotometer.* This apparatus likewise consists of three units, a spectrometer, a Martens photometer and auxiliary equipment for mounting and illuminating the samples. The arrangement is illustrated in Fig. 4, which is a reproduction of a diagram shown on page 13 of the Gaertner catalogue.[12]

The illumination unit consists of a white-lined diffusing sphere containing four 200-watt incandescent lamps. For transmission measurements

[11] This arrangement of optical parts in the Martens photometer is considered superior because no optical part, with reference particularly to the biprism with its inclined refracting surfaces, is placed between the wollaston and nicol prisms.

[12] Bulletin No. 126 entitled *Optical Instruments of Recent Design.*

(upper part of diagram) a diffusing white surface is placed at the back of the sphere and serves as the illuminant; if desired, this may be replaced by an opal glass plate permitting illumination from an external source such as the mercury arc. Light from the diffusing surface is collimated by a lens and then separated by a pair of rhombs into two beams which pass through the tubes holding solution and solvent. The beams are then directed by rhombs and lenses into the Martens photometer, as shown.

For reflectance measurements (lower part of diagram) the first pair of rhombs is removed, the two beams coming directly from the surfaces of sample and reference standard. The sample and standard are carried on a holder and may be interchanged in position.

The photometric field is of type II-*a*, with dividing line horizontal. To secure this, the photometric field of the Martens photometer is projected through the entrance slit to a position in the collimator objective where it is viewed from the ocular slit without eye-lens. Transmission and reflectance are computed by the same formulas as are used with the two instruments already described. The instrument also is made direct-reading if the angle of match can be secured at exactly 45°.

(3) Hilger spectrophotometric equipment

The most recent visual spectrophotometer made by Hilger is described in two papers by Dowell.[13] Fig. 5 is a reproduction of Fig. 3 from

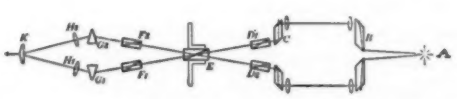

FIG. 5. Photometer designed by Dowell and used on the Hilger industrial and research spectrophotometer.

the first paper and illustrates the optical system employed.

Light from A is separated and collimated by rhombs and lenses as shown at B and C. The two beams pass through the respective fixed nicols, D_1 and D_2, the rotatable nicol, E, the fixed nicols, F_1 and F_2, the deflectors, G_1 and G_2, and the lenses, H_1 and H_2, to a biprism, K, mounted on the collimator slit of the spectrometer. The biprism edge is at right angles to the slit and forms the dividing line in the juxtaposed spectra, the photometric field being of type I. The planes of polarization of D_1 and D_2 are perpendicular to each other, as are also those of F_1 and F_2; the planes of polarization of D_1 and F_1 are parallel, as are also those of D_2 and F_2.

If d_1 and d_2 represent the amplitudes of the beams transmitted, respectively, by nicols D_1 and D_2 and if ϕ is the angle of rotation of nicol E required to equalize the intensities of the beams transmitted by nicols F_1 and F_2, then

$$d_1/d_2 = \tan^2 \phi$$

and the ratio of intensities is given by the expression

$$I_1/I_2 = (d_1/d_2)^2 = \tan^4 \phi.$$

From this, it follows that

$$\begin{aligned} T &= \tan^4 \phi_1 \cot^4 \phi_0 \\ &= \cot^4 \phi_2 \tan^4 \phi_0 \\ &= \tan^2 \phi_1 \cot^2 \phi_2, \end{aligned}$$

where T is the transmission of a sample inserted in either beam, ϕ_1 the angle of match for the sample in beam 1, ϕ_2 the angle of match for the sample in beam 2 and ϕ_0 the angle of match with the sample removed from both beams.[14]

The $\tan^4$ arrangement gives a more open scale for low transmissions than does a $\tan^2$ arrangement and is, therefore, considered especially suitable for absorption measurements. Other features of the arrangement stressed by Dowell are (1) no optical parts are placed between the polarizing prisms and (2) the nicols, F_1 and F_2, are effective in removing scattered light transmitted by nicol E.

In the assembled apparatus provision is made for reflectance measurements as well as for transmission measurements. For this purpose, the lamphouse carrying A and B, Fig. 5, is rotated 90° about a vertical axis between B and C. The arrangement is indicated in Fig. 6, left,

[13] J. H. Dowell, *A New Polarizing System for Spectrophotometers*, J. Sci. Inst. **8**, 382–384 (1931). *A New Industrial and Research Spectrophotometer*, J. Sci. Inst. **10**, 153–156 (1933). See also Hilger Publication No. 156/3, *Outfits for Absorption Spectrophotometry*, pp. 26–29, 1933.

[14] Compare the formulas given above for the König-Martens spectrophotometer. The final form in either case depends, of course, on whether the beam extinguished when θ or ϕ equals zero is defined as beam 1 or 2.

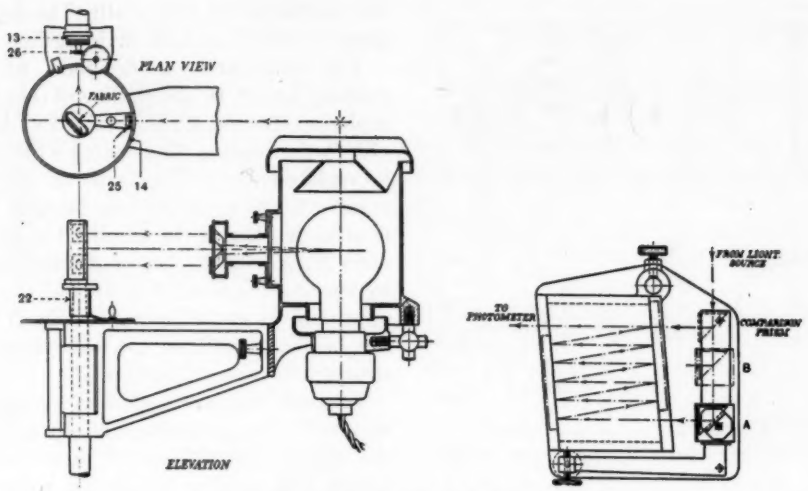

FIG. 6. Details of sample holders designed by Dowell.

which is a reproduction of Fig. 4 of Dowell's second paper. The end of the photometer box, *C*, in Fig. 5, is indicated at (13) in Fig. 6. The sample (fabric) and reference standard are mounted one above the other on a fitting (22) which is an extension of the spindle on which the lamphouse rotates. The angle at which the sample and standard are fixed for measurement is determined from the scale and index, (25) and (14). For reflectance measurements a 500 cp Point-o-lite lamp is used.

At the right in Fig. 6 is outlined the attachment used to measure the reflectance of polished surfaces. This consists of a platform carrying a prism and a mounting by means of which two plane reflecting surfaces of the material being tested are placed accurately parallel and facing each other. The light follows the course indicated, the comparison beam being reflected into the photometer by means of a right-angled prism underneath the platform. The purpose of this arrangement is to secure increased sensitivity by multiple reflection.

Dowell's instrument is considered a development from the Hilger-Nutting spectrophotometer. The most recent design of this latter instrument is described in Fig. H56a of the Hilger catalogue.[15]

[15] Section H, entitled, *Spectrophotometers, Colorimeters and Apparatus for Sensitometry*, p. H5, August, 1932.

2. Non-Polarization Methods

Various other means than polarization of light have been used in visual spectrophotometers to vary the intensity or brightness in a continuous and known manner. Such are rotating sectors variable while in motion, variable distance with the inverse square law, absorption wedges, variable voltage on lamps, etc. Some of these will now be illustrated.

(1) Keuffel and Esser color analyzer

The arrangement of this spectrophotometer is shown in Fig. 7.[16] As in some of the polarization instruments, the apparatus is made up of three principal parts—in this case, (1) an illuminating device with sample holders for transmission and reflectance measurements, (2) a rotating disk photometer and (3) a spectrometer.

The rotating disk photometer[17] consists of two sectored disks which revolve around the same axis and in the same direction, the speed being great enough to eliminate flicker. One sector is larger in diameter than the other and transmits a constant amount of the incident light near its

[16] This is a reproduction of Fig. 1 from a paper by C. W. Keuffel, *A Direct Reading Spectrophotometer*, J. O. S. A. and R. S. I. **11**, 403–410 (1925).

[17] For additional details the article by Keuffel should be consulted.

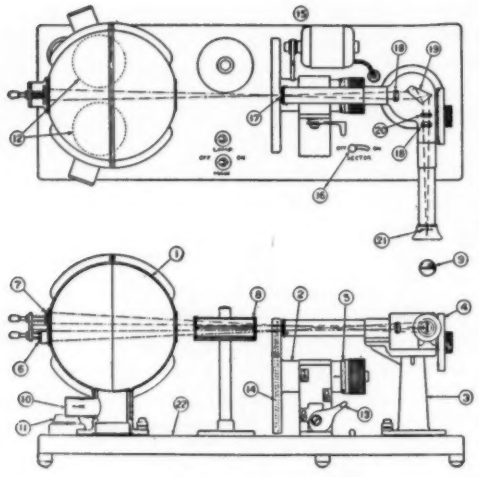

FIG. 7. Keuffel and Esser color analyzer. 1. Spherical light source. 2. Photometer. 3. Spectrometer. 4. Wave-length scale. 5. Photometer scale. 6. Holder for standard sample. 7. Holder for reflection samples. 8. Holder for transparent samples. 9. Field of view through eye slit. 10. To vacuum ventilator. 11. Plug for vacuum ventilator. 12. 400 watt lamps. 13. Lever for raising photometer. 14. Sector disks. 15. Universal 110 volt motor. 16. Speed control rheostat. 17. Entrance slit. 18. Collimator objectives. 19. Dispersion prism. 20. Bi-prism. 21. Eye slit. 22. Cast aluminum base.

edge when in rotation. The combined aperture of the two sectors near the edge of the smaller one may be varied relative to that of the constant sector by an amount varying from 0 to 110 percent; this variation is produced while the sectors are rotating by turning the knurled handle to which the photometer scale is attached.

The illuminating device consists of a white-lined sphere containing two 400-watt lamps. For reflectance measurements the sample and standard are placed, as shown, at the rear of this sphere, the sample above and the standard below. Direct illumination on sample and standard from the lamps is prevented by the use of shields. Light from the sample (7) passes through the constant aperture of the rotating sector, that from the standard (6) through the variable aperture, as shown. The directions of the axes of these beams are determined by the angle of the biprism (20), placed between the dispersing prism and the telescope lens. The edge of the biprism forms the dividing line of the circular photometric field and is viewed by the eye placed at the ocular slit. The instrument is thus of type II-*a*, with dividing line horizontal.

The instrument is designed to be direct reading. Let R_x be the reading of the sector when a photometric match is obtained with sample at (7), R_0 the reading for a match when the sample is replaced by a duplicate of the reference standard. The apparent reflectance of the sample relative to that of the standard is given by the ratio R_x/R_0. The value of R_0 may be adjusted by varying the relative widths of the lower and upper halves of the entrance slit. If this value can be made 100.0 at all wave-lengths the instrument becomes direct reading.

Transmissions are measured by replacing the sample (7) by a duplicate of (6) and mounting the sample in the upper beam between the sphere and sector. Absorption tubes are thus shown mounted at (8), the lower one being for the solvent. For low transmissions there is an adjustment (lever at (13)) which lowers the sectors, doubling the amount of the transmitted light and giving a more open scale, the sectors being so constructed that the scale then reads four times the actual value.

(2) Lummer-Brodhun spectrophotometer[18]

This instrument is outlined in Fig. 8. The illustration is a combination of Fig. 11 of the O. S. A. spectrophotometry committee report[19] and of Fig. 7 of a paper by Hyde.[20] The instrument is characterized by the separate collimators, C_1 and C_2, and the Lummer-Brodhun cube, L–B, by means of which the two beams are brought into proper juxtaposition. In the diagram as shown, the photometric field would be of type II-*c*. A more common type of field, known as the Lummer-Brodhun contrast field, is that illustrated in Fig. 1 as type II-*d*. When the two parts of the photometric field are matched, the two trapezoids are approximately 8 percent darker than the rest of the field because of the reflection from transparent non-absorbing glass

[18] O. Lummer and E. Brodhun, *Photometrische Untersuchungen, V, Ueber ein neues Spektralphotometer*, Zeits. f. Inst. **12**, 132–139 (1892). See also Schmidt and Haensch Catalog II, *Spektralphotometer*, p. 5, October, 1930.

[19] Reference 1.

[20] E. P. Hyde, *Slit-Width Corrections in Spectrophotometry and a New Form of Variable Sectored Disk*, Astrophys. J. **35**, 237–267 (1912).

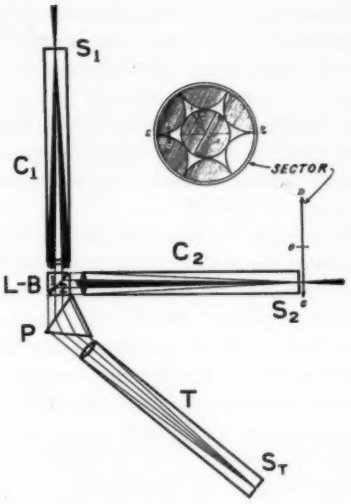

FIG. 8. Lummer-Brodhun spectrophotometer with variable rotating sector designed by Hyde.

plates properly placed in each beam close to the *L–B* cube. At low brightnesses this contrast type of field has greater sensibility than the equality-of-brightness field.

The Lummer-Brodhun spectrophotometer is particularly suitable for measuring the relative spectral energy distribution of two illuminants, because of the wide separation of the two entrant beams and has been used considerably for such measurements both with incandescent illuminants and with fluorescent materials. No means for varying the brightness of either half of the photometric field in a continuous and accurately known manner was originally provided on the instrument itself. The width of one of the collimator slits has, therefore, often been used to vary the brightness of the field, the brightness being taken proportional to the slit-width. This procedure is not to be recommended, however, unless the use of other more accurate methods is unnecessary.

To avoid this undesirable use of variable slits, any of the methods of varying brightness noted elsewhere in the paper may be used with the Lummer-Brodhun spectrophotometer. Rotating sectors of fixed or variable aperture (i.e., variable while in rotation) have often been used. Several have been designed that are similar to the *K* and *E* sector already described in that they consist of two sectors in series, rotating at the same rate, whose relative position may be varied while in motion.

Preceding these designs were those by Brodhun and Hyde. The Brodhun sector[21] consists of a pair of rotating rhombs which carry the beam of light across a stationary sector whose aperture may be varied while the rhombs are rotated. This sector is quiet and convenient but transmits only 0.4 of the incident light at its maximum opening. The scale is linear.

The sector designed by Hyde is illustrated in Fig. 8.[22] The rotating sector is moved bodily along the direction *C–D* in front of slit S_2, its effective transmission depending on its position relative to the slit. In this particular sector, the transmission varies from 0.90 at radial distance, *a*, to zero at radial distance, *b*, in this respect being superior to sectors which have radial openings limiting the maximum transmission to approximately 0.50. A second feature of this sector is the shape of the edges which are so designed that the transmission gradient, $\Delta T/T$, changes but slightly for values of T greater than 0.07. ΔT is the change in transmission corresponding to a given linear motion (e.g., 0.1 mm) along *C–D*.

A cylindrical sector has been recently designed[23] for use in a similar manner. This is illustrated in Fig. 9, which is a reproduction of

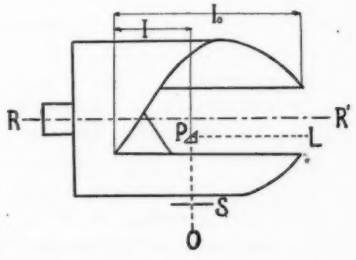

FIG. 9. Variable rotating sector designed by Dunn.

[21] E. Brodhun, *Messbare Lichtschwächung durch rotierende Prismen und ruhenden Sektor*, Zeits. f. Inst. **27**, 8–18 (1907). See also Schmidt and Haensch Catalog III, *Rotierende Sektoren und Sektorenmesseinrichtungen*, p. 4, June, 1929.

[22] Hyde's sector design has since been incorporated in the Lummer-Brodhun spectrophotometer assembly by Schmidt and Haensch. See Catalog II, p. 6.

[23] F. L. Dunn, *A Cylindrical Rotating Sector Photometer*, Rev. Sci. Inst. **2**, 807–809 (1931).

Fig. 2 of Dunn's paper. The direction of motion is along R–R'. The actual transmission of the rotating sector varies from zero to 1.00, the beam being brought along the direction, L–P, and directed into the slit, S, by the prism, P. In the design shown, the transmission is linearly proportional to distances along R–R' and is given by the ratio I/I_0.

(3) Spectrophotometric equipment at the National Physical Laboratory

Two designs of apparatus have been constructed at the National Physical Laboratory, both employing fixed rotating sectors of various apertures in combination with auxiliary devices for securing continuous variation of brightness.

(*a*) *Design by Guild.* This equipment is illustrated in Fig. 10, which is a reproduction of Fig. 2 of Guild's paper.[24] A constant deviation spectrometer is provided with a second collimator, C_2, and a combination prism, P, so mounted that a Lummer-Brodhun contrast field is secured of type II-*d*, although not exactly of the same pattern as that shown in Fig. 1.

Light from an incandescent lamp, S, with single vertical-coil filament, is formed into two beams by lenses and prisms and focussed onto the two collimator slits, as shown. The diagram shows the apparatus arranged for spectral transmission measurements, the sample being placed in beam 1. For spectral energy measurements, the prism, R_1, is removed and the standard and unknown illuminant alternately illuminate C_1, S serving as the comparison lamp.

Fixed aperture sectors, D_1 and D_2, placed in the beams, provide the major part of the necessary variations in brightness. Thirteen sectors are used, with effective transmissions varying from 0.90 to 0.010. In addition, a pair of absorption wedges are mounted at W to provide the necessary continuous adjustment of brightness within the intervals afforded by the sectors. In these wedges the absorption is due to finely divided lamp-black in gelatine. The wedge scale is calibrated in place by means of the rotating sectors, with the use of (1) a series of sectors at some one wave-length and (2) one chosen sector at the various wave-lengths. Thicknesses are, of course, correlated with the linear scale readings on the wedge unit, so that the wedges may be completely calibrated by following this procedure.

This design of instrument gives high light efficiency and the combination of sectors and wedge-unit makes possible the measurement of very low transmissions.

(*b*) *Design by Buckley and Brookes.* The arrangement of apparatus by Buckley and Brookes is as shown in Fig. 11, which is a reproduction of Fig. 1 of their paper.[25] In this case the Lummer-Brodhun cube, LB, is placed outside the spectrometer but is imaged by the lens, L_3, in the plane of L_4, giving a field of type II-b. Sectors of fixed aperture are placed at D_1 and D_2 as shown, giving discontinuous variations in brightness as on the previous apparatus.

Continuous variations in brightness are secured by varying the voltage on lamp A_1. Calibration of the scale is carried out by means of Wien's equation:

$$E_\lambda^S d\lambda = kC_1\lambda^{-5}e^{-C_2/\lambda\theta}d\lambda.$$

The symbols have the usual significance, except that S refers to the true temperature and θ to the color temperature. From this it is shown that

$$\log (E_\lambda^S/E_\lambda^{S_1}) = -(C_2/\lambda)(1/\theta - 1/\theta_1) = \log (E_\lambda^V/E_\lambda^{V_1}),$$

where S_1 and θ_1 refer to the temperatures at the standard voltage, V_1, and S and θ refer to any other voltage, V. If $-\log (E_\lambda^V/E_\lambda^{V_1})$ be designated as D_λ^V, then

$$D_\lambda^V = (C_2/\lambda)(1/\theta - 1/\theta_1)$$

and for any other wave-length, λ', it follows that

$$D_{\lambda'}^V = (\lambda/\lambda')D_\lambda^V.$$

Hence the relation between D_λ^V and voltage can be computed for all wave-lengths after it has been determined at some one wave-length. At that one wave-length, it is experimentally determined by means of the sectors.

In measuring transmissions, the sample is placed at F. Frosted glass is placed at O_1 and O_2, immediately in front of the gas-filled projection lamps, A_1 and A_2. At P is a prism which may be

[24] J. Guild, *An Equipment for Visual Spectrophotometry*, Trans. Opt. Soc. **26**, 74–94 (1924–25).

[25] H. Buckley and F. L. C. Brookes, *A New Type of Visual Spectrophotometer*, J. Sci. Inst. **7**, 305–317 (1930).

inserted to ascertain whether the introduction of the sample F has caused any important deviation of the beam. The image of O_2 is formed on a screen about 10 feet away and is magnified about 20 times. If, because of non-parallelism of the surfaces of F, the image is deviated, it may be brought back to its original position by movement of the lens L_2 in its own plane. Errors due to multiple reflections between LB and F are eliminated by inclining LB sufficiently to throw the image of O_2 about 5 mm off the slit, C.

III. Factors Affecting the Reliability of Spectrophotometric Data

1. Illumination and Reference Standards

The use of the substitution method in spectrophotometry may be considered an absolute necessity for accurate work. In this method, the path of the light without and within the instrument should be effectively identical for the sample being measured and for the standard with which it is compared. As already illustrated in the various types of instrument, this may be accomplished either by having the light from the sample and standard brought successively into one-half of the photometric field, the two light beams being evaluated in turn by means of the light in the comparison beam, or by having the light beams from the sample and standard simultaneously traverse different paths leading to the two parts of the photometric field and then interchanging them.

(1) For transmission measurements

In measuring the transmission of a transparent non-diffusing sample such as a glass filter, the standard is merely the "blank" beam.[26] The important thing to consider in this case is whether or not the unavoidable change in optical path caused by introduction of the sample into the beam has effectively changed the standard; in other words, has the effective illuminant been varied in any respect by insertion of the sample being measured. Such variation may occur by change of optical distance in a diverging beam, by displacement of a focussed image of a non-uniformly bright illuminant (or by any change in the used area of such illuminant) or by multiple reflections between the surfaces of the sample and the optical parts of the apparatus.

One should also consider whether the average distance traversed by the used portion of the beam through the sample is significantly greater than the thickness of the sample and whether the variations of transmission with changes in temperature, which occur at certain wavelengths for nearly all materials, due to variations in room temperature or to heating of the sample in the beam, are of importance in any particular instance.

Methods for eliminating errors from certain of these causes have been noted in connection with some of the instruments described. It is not attempted here to discuss the various other ways by which such errors may be avoided. It is sufficient to say that accurate results cannot be claimed unless these factors have been properly considered.

Since nearly all materials whose spectral transmission is desired are transparent, non-diffusing substances, such, for example, as glass filters or dye solutions, the angular distribution of the incident illumination is relatively unimportant, provided it properly fills the photometric field. The sample is placed at right angles to the unidirectional beam and the same values are obtained as if the sample were diffusely illuminated. If, however, the sample scatters the incident light to any important extent, as does, for example, a photographic film or opalescent glass, the type of illumination and direction of transmitted light become as significant as the corresponding quantities in reflectance measurements.

(2) For reflectance measurements

The accurate measurement of the absolute reflectance of diffusing materials is very difficult. It has, therefore, become customary to measure the spectral reflectance of diffusing materials relative to that of some other material, designated as the reference standard, whose reflectance characteristics are considered known, or at least reproducible. Magnesium carbonate

[26] In case the transmittancy of a solution is to be measured, a cell containing the solvent is placed in the blank beam to compensate for losses by reflection and absorption of the solvent and end plates, this cell being similar in all respects to that containing the solution.

and magnesium oxide have both been widely used for this purpose. The use of the former should be discouraged, however, since it is now known to be subject to variations in reflectance amounting to several percent.

The advantages of a MgO surface as a standard of reflectance, when properly prepared, are as follows:[27] (1) It is nearly as good a diffusor as any other surface; (2) Its light reflectance of about 0.97 is nearly as high as that of any other surface; (3) The reflectance varies with wavelength in the visible spectrum by less than one percent; (4) Its reflectance is nearly, if not perfectly, constant with time; (5) The apparent reflectance for 45-degree incidence and normal viewing or *vice versa* is 1.00.

The disadvantage of MgO is its fragility. If desired, therefore, a working standard of porcelain, milk glass or other suitable material may be used; this may then be calibrated in terms of MgO and checked as often as desired.

Various types of illumination for reflectance measurements have been illustrated in the instruments described. It should be remembered, however, that no one particular condition of illumination and viewing will give complete information regarding the spectral reflectance of diffusing materials. Such information can be obtained only by measuring the apparent reflectance at several angles of view from 0° to 90° and for several angles of unidirectional illumination from 0° to 180° in the plane of view and from 0° to 90° in a plane at right angles to the plane of view. If the surface has structural regularities, it must also be rotated from 0° to 90° in its own plane.[28]

If one is limited to a single set of conditions, it is a matter of choice which to adopt. Two sets of conditions are worthy of special notice:

1. Diffuse illumination, perpendicular viewing. This is illustrated in Figs. 2, 4 and 7. One may distinguish between illumination which is in effect completely diffused, i.e., of equal intensity from all directions over the solid angle 2π, Fig. 2, and illumination which only approximates this condition, Figs. 4 and 7, the approximation depending on the reflectance and diffusion of the sphere wall. The features of completely diffused illumination and normal viewing are: (1) The reflected light is unpolarized, unless the sample has certain types of structural regularity; (2) The apparent reflectance with diffused illumination is numerically equal to the reflectance with unidirectional illumination at 90°; (3) The regular component of reflectance is included; (4) The sample is uniformly illuminated.

2. Unidirectional illumination at 45°, angle of view 90°. This set of conditions was adopted by the International Commission on Illumination in 1931.[29] The following points may be noted with reference to this method of illuminating and viewing: (1) The apparent reflectance obtained is numerically equal to that obtained with illumination at 90° and 45° viewing. This latter condition, however, gives a greater brightness to the reflecting material; a uniform brightness is also more readily obtainable in the latter case. (2) The reflected light is, in general, partially polarized. (3) The regular component of reflectance is excluded.

For either set of conditions, it is essential that the sample and standard receive identical illumination. It has been pointed out[30] that this condition is not fulfilled if the sample and standard are alternately placed in position at the opening of a diffuse-illumination device; in this case, because of multiple reflections, the sample and standard are not subjected to identical illumination, even if identically placed, unless they are of equal reflectance. The error is small unless the sample and standard differ considerably. The proper condition is to have both sample and standard illuminated throughout the measurements, as illustrated in Figs. 2, 4 and 7.

[27] *Preparation and Colorimetric Properties of a Magnesium-Oxide Reflectance Standard*, Bureau of Standards Letter Circular, LC 395, December 9, 1933. References to the authorities for the advantages here given may be found in the circular.

[28] Apparatus enabling such universal measurements to be made, in addition to measurements at various angles of view with completely diffused illumination, for both reflectance and transmission, either as a function of wavelength or as averaged with respect to the total luminous effect, has been constructed at the Bureau of Standards and is described in a paper by H. J. McNicholas, Bur. Standards J. Research, in press.

[29] Commission Internationale de l'Eclairage, Compte rendu des seances, 1931, p. 23.

[30] A. C. Hardy and O. W. Pineo, *The Errors due to the Finite Size of Holes and Sample in Integrating Spheres*, J. Opt. Soc. Am. **21**, 502–506 (1931).

In case the reflected light is partially polarized, the plane of polarization should be so oriented as to be at 45° to the plane of polarization of the dispersing prism or other polarizing parts of the apparatus. The polarized component is then transmitted as if unpolarized and no error results.

(3) For relative energy measurements

The standard in this case is usually an incandescent lamp of known relative spectral energy distribution. Over the range covered by visual measurements, this standard can be most accurately established by way of its color temperature. If the color temperature of the lamp is accurately known, its spectral energy distribution may be obtained from various published tables of spectral energy distribution derived on the basis of Planck's equation.[31]

Such relative energy measurements are now rarely attempted on incandescent illuminants in the visible spectrum, because of the easy and accurate means of deriving such values via color temperature. In case, however, the energy distribution in other types of illuminants, e.g., fluorescent spectra, is desired, such a standard becomes necessary.

2. Wave-Length Calibration

A thorough calibration or check of the wave-length scale of the spectrophotometer should always be made, regardless of whether or not the instrument reads directly in wave-length. Such calibration can be made by means of the mercury and helium lamps with such additional wave-lengths as may seem necessary or desirable. Furthermore, constant check should be kept on the calibration, particularly if any but the best bilateral slits are used. It is good practice to make such check every time one uses the instrument or changes the slit-widths, unless experience has shown that the stability is such as to make this unnecessary.

3. Stray Light

Stray light may introduce serious errors in spectrophotometric measurements if not eliminated, not only in the end regions of the visible spectrum but also in regions of low transmission for certain samples which transmit freely in adjacent regions. This should be eliminated as far as possible by keeping all optical surfaces carefully cleaned and by the use of optical filters in the regions of low luminosity.

The proper type of filter to use is one which transmits freely in the wave-length region where measurements are being made and absorbs strongly in all spectral regions of higher luminosity. A set of two or three for each of the blue and red ends of the spectrum is desirable, with special filters as needed for special purposes. A partial list of such filters has been published.[32]

4. Slit-Width Errors

The theory of slit-width corrections in visual spectrophotometry has been discussed by Hyde,[33] who develops an expression for the luminosity of a pure spectrum $\phi(\theta)$, in terms of that of the impure spectrum $F(\theta)$, resulting from the use of finite slit-widths, θ being the angle of deviation of the ray considered. The expression is:

$$\phi(\theta)=(1/A)[F(\theta)-K\Delta^2F(\theta)+L\Delta^4F(\theta)+\cdots],$$

in which the values of A, K and L depend only on the relative values of the collimator and telescope slit-widths and $\Delta^2F(\theta)$ and $\Delta^4F(\theta)$ are defined as follows:

$$\Delta^2F(\theta)=F(\theta+c)+F(\theta-c)-2F(\theta),$$
$$\Delta^4F(\theta)=\Delta^2F(\theta+c)+\Delta^2F(\theta-c)-2\Delta^2F(\theta),$$

in which c equals one-half the sum of the collimator and telescope slit-widths, expressed in the same units as θ.

Hyde illustrates the application of the formula to the case of spectral energy distribution measurements, with numerical examples of the magnitudes involved. For spectral transmission measurements, the above expression may represent the luminosities with no sample in the beam. If a sample whose transmission is T is placed in the beam and $\phi_T(\theta)$ and $F_T(\theta)$ represent the corresponding luminosities, we have

$$T=\frac{\phi_T(\theta)}{\phi(\theta)}=\frac{F_T(\theta)-K\Delta^2F_T(\theta)+L\Delta^4F_T(\theta)+\cdots}{F(\theta)-K\Delta^2F(\theta)+L\Delta^4F(\theta)+\cdots},$$

[31] M. K. Frehafer and C. L. Snow, Bur. Standards Misc. Pub. No. 56, 1925; J. F. Skogland, Bur. Standards Misc. Pub. No. 86, 1929; R. Davis and K. S. Gibson, Bur. Standards Misc. Pub. No. 114, Table 2, 1931.

[32] J. Opt. Soc. Am. **21**, 570, Table II (1931).

[33] See reference 20.

assuming the relative values of the slit-widths to be unchanged in the two cases. For a given sum of collimator and telescope slit-widths, the errors or corrections are least when the two slit-widths are equal. For this case

$$K=1/12 \quad \text{and} \quad L=1/90.$$

The application of the formula is simplified if the Δ^4 terms are neglected. This is probably justified in most cases, not only because of the relative magnitudes of K and L but because the correction is, of necessity, in most cases, only of an approximate nature. This approximation arises, not from inaccuracy of the formula, but because of uncertainty in the luminosity function assumed in order to evaluate the Δ^2 and Δ^4 terms. The exact values of $F(\theta)$ and $F_T(\theta)$ which should be used in the formula depend on (1) the observer's visibility function, (2) the relative energy distribution of the illuminant, (3) the dispersion and transmission of the instrument—all as functions of wave-length. The observer's visibility is, in general, unknown and in any case depends on the size and brightness of the photometric field. It is customary, therefore, to assume an average visibility function and to determine or estimate the other quantities.

If the Δ^4 terms be neglected, the formula becomes

$$T=\frac{\phi_T(\theta)}{\phi(\theta)}=\frac{F_T(\theta)-K\Delta^2F_T(\theta)}{F(\theta)-K\Delta^2F(\theta)}.$$

It may be noted that: (1) The errors or corrections are zero when T is constant over the range of wave-lengths included by $\theta\pm c$, that is, provided $F_T(\theta)=kF(\theta)$ over that range, k being a constant. (2) They are also zero when c is zero. With incandescent illuminant this can strictly be attained only in the limiting case of zero slit-widths. With homogeneous light, however, as from the Hg or He lamps where only a single line is transmitted by the slits, c is in effect equal to zero. (3) The errors or corrections are approximately proportional to the square of the sum of the ocular and telescope slit-widths. For, expanding $F(\theta+c)$ and $F(\theta-c)$ by Taylor's theorem, we have

$$\Delta^2F(\theta)=F''(\theta)c^2,$$

neglecting higher even-power terms. (4) They also, therefore, depend on the curvature of that portion of the luminosity curve included by the slits, and not on the slope.

The importance of the slit-width errors and the need for making corrections depend on the slit-widths used and the purpose for which the data are obtained. Because of the labor of making the corrections, it is desirable, if possible, to use such narrow slits that the errors are negligible. It should be noted also that slit-width errors or corrections can be determined experimentally. Since the error is proportional to the square of the slit-widths, the measured values of transmission rapidly approach a constant limiting value as c is decreased. Extrapolation to zero width can, therefore, readily be made, provided the slit-width errors are not too small relative to the photometric uncertainty.

5. Checking the Reliability of the Photometric Scale

Two relations may be accepted almost without question in visual photometry, *viz.*, the inverse-square law and Talbot's law of the rotating sector. The former law is one of the fundamental relations of photometry but is rarely used directly in spectrophotometry. In order that the law be accurately obeyed, the illuminant cannot be operated too close to the diffusing surface. The resulting brightness in the field of the spectrophotometer is too low for general utility, although the method affords a valuable means of checking the photometric scale in the brighter parts of the spectrum. The law applies strictly to a point source. However, if an extended source is used, concentrated essentially in a plane at right angles to the direction of the beam, and the minimum distance used is 30 times the diameter of the source, the errors in the inverse-square law should not exceed 0.1 percent.

Talbot's law states that when a beam of light is periodically interrupted, as by a rotating sectored disk of aperture α degrees at a frequency great enough to eliminate flicker, the eye integrates the transmitted light in such a way that apparent intensity is equal to $\alpha/360$ of the original intensity. Provided the angular aperture is accurately known, the relation is perhaps the most reliable of any available for maintaining or checking the accuracy of the photometric

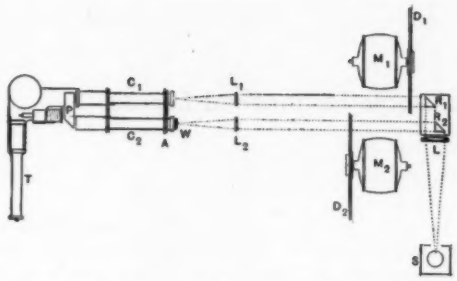

FIG. 10. Spectrophotometric equipment designed by Guild at the National Physical Laboratory.

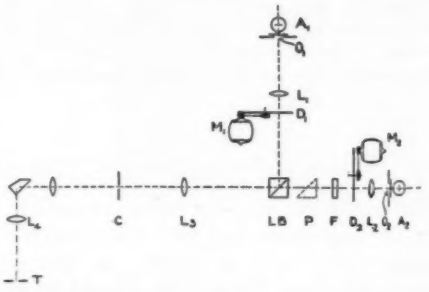

FIG. 11. Spectrophotometric equipment designed by Buckley and Brookes at the National Physical Laboratory.

scale. The method is particularly suitable for checking the reliability of polarization instruments, as illustrated in Fig. 2, or for extending the scale to low values of transmission while maintaining its reliability, as illustrated in Figs. 2, 10 and 11. In all these cases rotating sectors of fixed aperture are used, which may be accurately calibrated mechanically. Rotating sectors of variable aperture should be checked for accuracy, the same as polarization devices, optical wedges, etc.

It should be emphasized, however, that the reliability of the photometric scale is only one of several factors affecting the accuracy of spectrophotometric measurements; the others have been noted above. All the factors must be considered in order to be certain that accuracy is obtained. A closing check on the reliability of spectrophotometric data may be secured by making the same measurements on different apparatus or by using filters of accurately known transmission.[34] Agreement of results will usually indicate reliability. Measurements on a solution at different thicknesses will also assist in detecting errors, the negative logarithm of the transmittancy being accurately proportional to the thickness.

6. Precision of Measurement

While precision of setting or reproducibility of results is no guarantee of reliability, it is a matter of considerable importance. The eye is capable of remarkable precision when the proper equipment and technique are employed and can maintain this precision without undue strain for considerable lengths of time when properly used. The following factors contribute notably to the precision of spectrophotometric measurements: (1) comfort of the observer, (2) ease of manipulation of the photometric device, (3) large and uniform photometric field, (4) relatively high field brightness,[35] (5) fineness of dividing lines, (6) freedom from stray light. If these conditions are properly satisfied average deviations from the mean of only a fraction of one percent are readily obtainable.

[34] For example, see J. Opt. Soc. Am. **24**, 58, paper No. 14, 1934.

[35] Brightness can rarely be obtained too high in spectrophotometry. Room illumination, extraneous light, etc., also affect the precision of measurement. In general, the room illumination should be low but the observer's eyes should be totally shielded from all other light only at very low brightnesses of photometric field.

A Quantitative Optical Test for Telescope Mirrors and Lenses

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(Received June 19, 1934)

1. Light from an optical slit is brought to a focus by the mirror or lens under test. A single tensioned wire of about 0.003 to 0.006 inch diameter is placed just in front of focus and arranged symmetrically on the axis of the returning cone parallel with the optical slit. **2.** The images of two parallel gauge slits, illuminated with a contrasting color, are symmetrically superimposed on the speculum image by means of an unsilvered diagonal or equivalent. The gauge slits are also arranged to look parallel to the optical slit image. **3.** The eye, when placed at the focus of the returning cone, sees the shadow image of the test wire and the image of the gauge slits against the illuminated speculum. **4.** By moving the test wire along the axis of the returning cone, its shadow image is expanded and made to touch or intersect the parallel gauge images at the various zonal radii of the mirror under test. The motion of the wire is a direct measure of the aberration existing between those respective zonal radii. **5.** The geometry of the test is described and a few sketches show the appearances of some of the common forms of aberration.

SEVERAL methods have been proposed and used in the testing of optical surfaces. Among these the Foucault knife-edge test is the most popular as it is visual and requires only simple apparatus. The Hartman test, being photographic, is used where a final analysis of the performance of an optical instrument is desired. The interference methods that have been proposed are very accurate but, requiring complicated apparatus and laborious readings, have not proven popular.

About 1923, Vasco Ronchi proposed a method of testing, which has the appearance of an interference method but involves only simple apparatus.[1] J. A. Anderson and R. W. Porter have found the Ronchi test to be fully as sensitive as the Foucault test and, under favorable conditions, capable of showing an error as small as 1/10 of a wave-length in the wave surface leaving the principal plane of the instrument.[2] However, the Ronchi test is only qualitative as it does not indicate the amount of aberration present.

While considering the theory of the Ronchi test, and wishing that it were also quantitative, the writer discovered a scheme using this principle with the addition of very simple auxiliary apparatus, which resulted in a quantitative test.

QUANTITATIVE FEATURE OF TEST

The feature, making the test quantitative, consists in superimposing two fine parallel line images, which serve as gauges, on the lens or mirror under test. The Ronchi grating is reduced to a single wire aligned parallel with the gauge lines and the optical slit. By moving the wire axially its shadow image is symmetrically expanded until it crosses the fixed gauge images at the zonal radii under test, the motion necessary being a measure of the aberration existing between these zonal radii.

The exact method of forming the line images on the mirror or lens is not important so long as it does not introduce errors larger than those of measurement. The method that first occurred to the writer was to stretch piano wires immediately in front of the speculum and view them by the diffraction around their edges or illuminate them by a light of contrasting color from one side. In the case of testing mirrors where the back (not the optical surface) of the mirror is polished and reasonably plane, a cardboard screen, placed against the back of the mirror, having the gauge slits and markings indicating the zonal radii cut in it and illuminated from behind, would be very simple and perhaps could not be surpassed. The method to be described produces virtual gauge line images against the speculum by the use of a small glass diagonal and has the advantage, that it is more

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[1] Vasco Ronchi, *La prova dei sistema ottici*, Bologna, 1925.

[2] Anderson and Porter, Astrophys. J. **70**, 175 (1929).

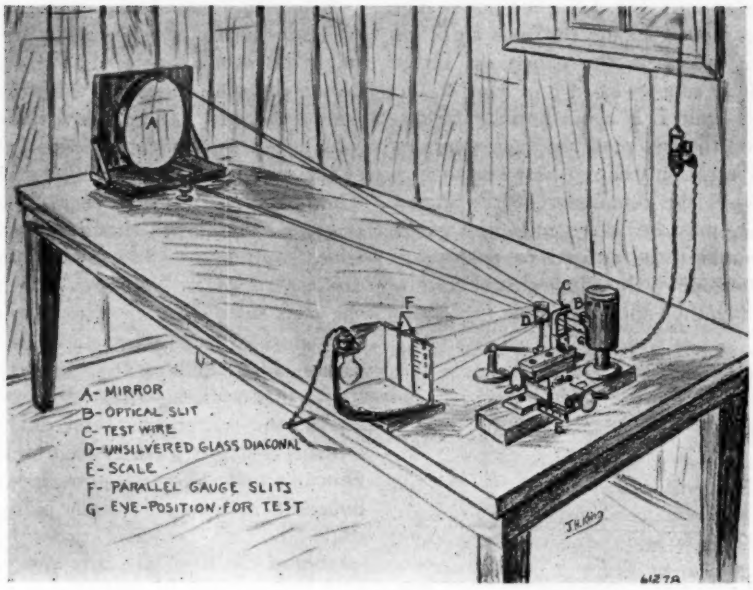

FIG. 1.

generally applicable especially where the lens or mirror may be mounted in an inaccessible place. The distortion produced by the use of a small parallel glass diagonal in having the cone of rays from the speculum pass through it is probably small, since the diagonal need be nothing larger than a microscope slide cover glass and can be placed very close to the eye. It may even be placed behind the test wire right next to the eye.

APPARATUS AND PROCEDURE

The set-up of apparatus is shown in the sketch of Fig. 1. Here we have at *B* an optical slit from which the rays of light emanate, strike the speculum *A* (under test) and return to focus at *G*, the position of the eye for the test. Immediately in front of *G* is placed a tensioned wire *C*, arranged symmetrically in the returning cone and parallel with the optical slit. (The writer has used a wire about 0.006 inch in diameter.) An unsilvered diagonal *D* reflects the image of two parallel gauge slits *F*, allowing them to be viewed at *G*, and appear symmetrically superimposed on the speculum *A* along with the shadow image of the test wire. The scale *E* is used in measuring the displacement of the wire *C* along the axis of the returning cone.

The procedure consists of adjusting the gauge slits until they appear as separated about $\frac{1}{4}$ or $\frac{1}{3}$ the diameter of the speculum and then leaving them fixed. The test wire is then adjusted along the axis until its shadow image appears to just touch the gauge images as in *a* of Fig. 2. The

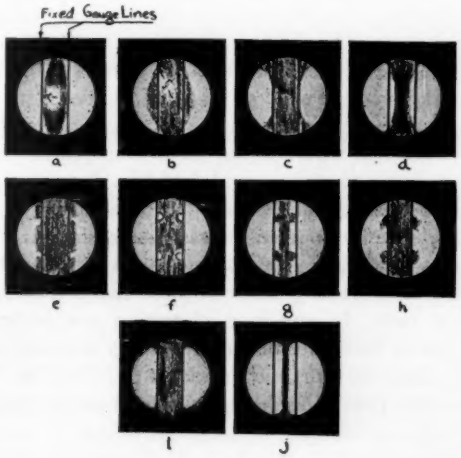

FIG. 2.

reading on the scale E, Fig. 1, is then noted. The wire is again moved along the axis until its shadow image intersects the gauge slit image symmetrically at zone r_2 as in b of Fig. 2. The reading on the scale E is again noted. The difference between the two scale readings measures directly the aberration existing between zone r_1 and r_2. Likewise, the remainder of the mirror outside zone r_1 may be tested with the gauge slits fixed. Should it be desired to test quantitatively inside zone r_1, the gauge slits should be moved closer together and the above procedure repeated.

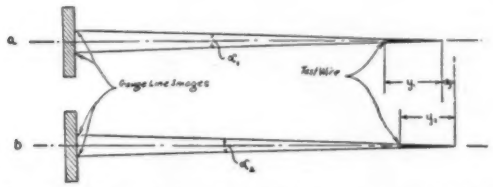

FIG. 3. Horizontal plan views—looking down on test from above.

GEOMETRY OF TEST

Fig. 3, a, shows a horizontal plan view (not the conventional vertical section through an optical system) of the test when adjustment is made for zone r_1 as in a of Fig. 2 and shows only the rays proceeding from zone r_1 to focus. Similarly, b of Fig. 3 shows the plan view of the adjustment for zone r_2. In a of Fig. 3, y_1 is the distance that the test wire C of Fig. 1 is situated in front of focus to appear to just touch the gauge slit images. The distance that the wire C, Fig. 1, is situated in front of focus for zone r_2 is designated by y_2, the shadow image of the test wire C appearing, in this case, as in b of Fig. 2. Δy is the distance test wire C is moved to effect these two conditions. Since in a and b of Fig. 3, angles α_1 and α_2 are equal (within limits affecting this test), it follows from substantially similar triangles that $y_1 = y_2$ and Δy measures directly the aberration existing between these two respective zonal radii. In the case of testing a spherical mirror at its center of curvature, Δy is equal to two times the difference in radius of curvature of the zones providing the optical slit remains fixed.

Before discussing the appearance of some common forms of aberration in this test, it might be well to state, that the shadow image is not quite as shown in the sketches. With a wide optical slit, the shadow image of the test wire is not sharp on the edges, while a narrow slit causes secondary diffraction effects to appear. The fine slit adjustment seems preferable, since it gives the sharper shadow image to the test wire. Too fine a test wire (0.001 inch diameter) does not apparently improve the sensitivity of the test, since then the wire has to be brought very close to focus to expand its shadow appreciably, again making it difficult to preserve a sharp shadow image. However, in spite of the above, the writer has been able to duplicate measurements at least as accurately as with the Foucault test. The test is much faster than the Foucault test when the latter is used with zonal stops and at the same time the qualitative feature of the Ronchi test is always observable.

APPEARANCES OF COMMON FORMS OF ABERRATION

In c–d, e–f, g–h and i–j of Fig. 2 are shown some common forms of aberration as they appear in this test. Aberration of the opposite sign to a–b of Fig. 2 is shown in c–d, the former being the case of outer zones too long in focus, and the latter, outer zones too short. The appearance of a raised zone is shown in e–f of Fig. 2, and g–h shows a depressed zone at the same place on the speculum. Turned edge, one of the worst enemies in the figuring of optical surfaces, is easily detected and can be accurately analyzed by this test. In all other tests the wire C, Fig. 1, has been kept just inside of focus but in the case of turned edge the sketches Fig. 2 i–j show the appearance of placing the wire just outside the focus as this gives more prominence to the effect. In quantitatively testing parabolic mirrors of short focal ratio at the average center of curvature, the Foucault shadows become very deep, necessitating the use of a large number of zonal stops. The present test would seem more suitable in such cases since the use of stops is eliminated and photometry is unnecessary.

A Grating Interferometer

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(Received May 21, 1934)

Discussions of the Michelson interferometer indicate that fringes are formed by the superposition of two approximately parallel wave fronts. Wave fronts having the necessary characteristics can be obtained by mounting gratings on the arms of the interferometer in the places of the customary reflecting mirrors. The construction and action of such an interferometer has been studied; it is here described. The instrument becomes its own monochromator when the gratings are rotated to positions of minimum deviation for some particular wave-length. The customary form of the Michelson interferometer may be considered as the special case of the grating interferometer for which the grating space is large in comparison with λ.

INTRODUCTION

THE relation which specifies the positions of maxima in the spectra produced by optical devices of the grating type is

$$\sin i = m\lambda - \sin i', \tag{1}$$

i being the angle of incidence, i' the angle of diffraction, m the order of the spectrum, λ the wave-length, and σ the grating space. The formula is applicable to transmission and reflection instruments alike. If $m=0$,

$$\sin i = -\sin i'. \tag{2}$$

Eq. (2) states that the maximum of the zero order is located at the position specified by the fundamental laws of reflection. The laws of reflection will specify the position of maxima corresponding to all values of m, however, if the grating space is large in comparison with λ. Since the edges of a mirror affect the incident wave front in the same way as the rulings of a grating, the laws of reflection are rigorously applicable when the incident beam is normal to the mirror, and when the mirror is infinitely extended in the plane determined by the incident beam and the normal to the mirror at the point of incidence. The laws of reflection can be regarded as special cases of the grating formula, for there is no essential distinction between an image reflected from a mirror and the maxima produced by a grating. The positions of both are governed by the same formula (1). Consequently, it is possible to substitute gratings for mirrors in all sorts of optical devices. Whether reflection or transmission gratings should be used in any particular case depends upon whether the grating is to supplant a reflecting or a transmissively silvered mirror. Many novelties would result from the substitution, and practical advantages might be obtained in certain cases. The present study examines the modifications of construction and action which the substitution would effect in an optical arrangement of the Michelson type.

THEORY OF THE GRATING INTERFEROMETER

The fringes produced by the interferometer are easily analyzed if the gratings are illuminated by parallel light. The intensity distribution for the arrangement is proportional to the square of the vector sum of the amplitudes of the two beams:

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\gamma. \tag{3}$$

A_1 and A_2 (the respective amplitudes which are superposed at the half-silvered mirror) have values which are expressible algebraically by the formula used for gratings illuminated with parallel light.

$$A_1 = (a_1c_1)\frac{\sin \pi a_1\theta_1}{\pi a_1\theta_1}\cdot\frac{\sin N_1\pi\sigma_1\theta_1}{\sin \pi\sigma_1\theta_1} \tag{4}$$

and A_2 may be expressed similarly.[1] Thus the instrument becomes its own monochromator when the gratings are rotated in such a way as to

[1] The symbols in this equation have the significance specified for them in W. L. Taylor, *College Manual of Optics*, Ginn and Company (1924), p. 55.

superpose the chosen maxima in the grating spectra. The principal maxima of each grating are given by (1). In order to superpose the chosen maxima at the transmissively silvered mirror, it is convenient to have the diffracted beam traverse the path of the incident beam in an identically reverse direction. In this case, $i=i'$; or

$$\sin\frac{(i+i')}{2}=\frac{m\lambda}{2\sigma}\sec\frac{(i-i')}{2}. \tag{5}$$

The angle $(i+i')$ is the angle of deviation, and (5) shows it to be a minimum when $i=i'$. The gratings will provide beams suitable for interferential purposes, therefore, when they are in positions of minimum deviation.

The position of the Michelson reflecting mirrors when producing infinity (circular) fringes follows from this relation; for, in the position of minimum deviation,

$$\sin i=m\lambda/2\sigma. \tag{6}$$

For the Michelson mirrors, σ is large as compared with λ. That is, for the unruled Michelson mirrors, the angles of incidence must be zero.

Description and Adjustment of the Grating Interferometer

In the following description of the apparatus which has been constructed and employed, it will be convenient to note three distinct features: (I) the manner of mounting the grating, (II) the methods of adjusting the interferometer, and (III) the simplifications which are allowable in the design of the instrument.

I.

A precise adjustment of the gratings in positions of minimum deviation requires the use of mountings which supply four axes of rotation. Three of these rotations are those which are commonly employed in adjusting a grating for use with a spectrometer. The fourth is introduced to make the rulings of the two gratings parallel. The details of construction are shown in Fig. 1. Of the four axes of rotation, only one is fixed in direction: it is the axis which is parallel to the arm of the interferometer. The four axes have a common intersection.

In order to effect gross rotation about the fixed axis, it was necessary to construct the arm of the interferometer in two parts; and these two parts were connected by a bearing which permitted rotation about the axis. A collar was screwed to that portion of the interferometer arm which was rigidly fixed to the base of the instrument. This collar and a cylindrical shell fitted into it comprised the indispensable mechanisms of the bearing. Since the inner diameter of the cylindrical shell was approximately four centimeters, the bearing did not obstruct the passage of light between the half-silvered mirror and the grating. The cylindrical shell was held in place by a screw ring on the side of the collar adjacent to the half-silvered mirror. A bracket, attached to the opposite end of the cylindrical shell, supported the additional part of the interferometer arm.

This portion of the interferometer arm was fitted with a turntable which could be rotated about an axis perpendicular to the fixed axis of the instrument. The rotation of the table could be accomplished by means of a slow motion screw. The threads of the screw could be disengaged, however, by a small lever; and the turntable was then free to rotate through large angles. A U-shaped standard, fixed to the turntable, carried a clamp which held the grating. The clamp was suspended between the jaws of the U-shaped standard, and could be rotated between them about an axis perpendicular to that of the turntable. This rotation was controlled by two screws which allowed either fine or gross adjustments. The axis about which this rotation occurred may be called the axis of reflection; for rotation about it produced effects which are described directly by the laws of reflection if the rulings of the grating are parallel to the axis of the turntable. The clamp which was suspended between the jaws of the U-shaped standard could be rotated about an axis perpendicular to the axis of reflection. This rotation was designed to control the orientation of the rulings of the grating with respect to the axis of reflection. The axis will be designated as the axis of diffraction. There was no fixed relation between the four axes other than those which have been mentioned.

A mounting for the grating which provides four axes of rotation is ideal because it allows each grating to be completely adjusted independently of the other. Such a mounting will be

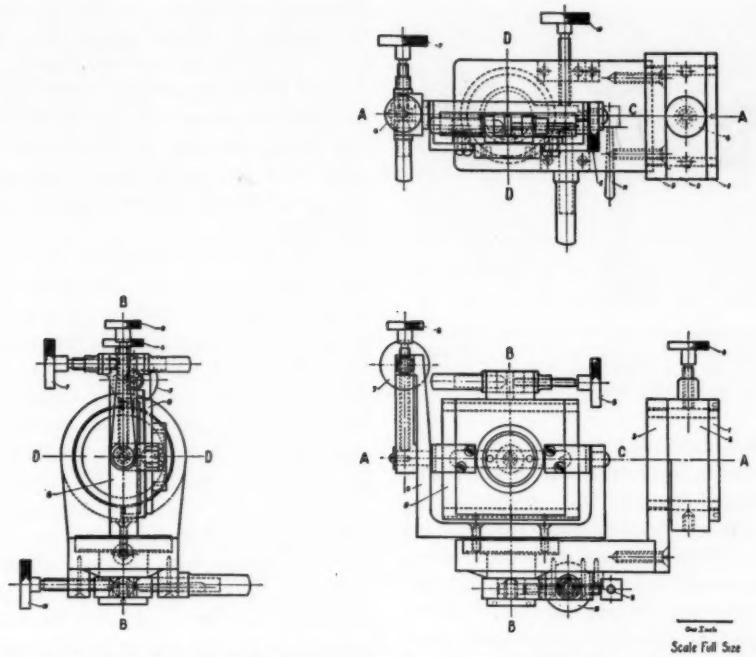

FIG. 1. Assembly drawing of ideal mounting. *AA*, fixed axis; *BB*, axis of turntable; *AC*, axis of reflection; *DD*, axis of diffraction; 1, screw ring; 2, collar and cylindrical shell; 3, bracket; 4, screw release for rotation about fixed axis; 5, screw for slow motion about axis of diffraction; 6, screw release for gross motion about axis of reflection; 7, screw for slow motion about axis of reflection; 8, U-shaped standard; 9, clamp for grating; 10, screw for slow motion about turntable axis; 11, lever release for gross motion about turntable axis.

called an ideal mounting. It will be seen later that ideal mountings are not necessary for the production of interference systems, for fringes can be produced when simplified mountings are used. The simplifications are obtained by eliminating one or more of the four axes. For instance, if the mounting did not permit rotation about the fixed axis, the grating would still have all the possibilities of adjustment supplied by a spectrometer table. A mounting of this type will be called, accordingly, a spectrometer mounting of the grating. Even in the adjustment of a grating for use with a spectrometer, however, four axes of rotation are required. In the spectrometer, one rotation is the motion of the slit about the collimator axis. In the grating interferometer, both gratings must be illuminated by the same slit. Rotation about the collimator axis will make the slit parallel to the rulings of one grating, which needs, therefore, merely a spectrometer mounting. The other grating must be supplied with an ideal mounting if the rulings of the two gratings are to be parallel to each other and to the slit. Fig. 2 is a grating interferometer in which one grating is supplied with an ideal mounting and the other with a spectrometer mounting.

II.

The interferometer is readily adjusted when it is fitted with a collimator and a telescope, both accessories being adjusted for infinity. Light from the source is made parallel before it is incident upon the half-silvered plate by passing through the collimator; and the images from the collimator slit, after diffraction at the gratings, pass from the half-silvered plate into the telescope and are brought to a focus at the cross hairs. Three slightly different procedures may be employed in adjusting the grating interferometer. They are the method of the Littrow mounting, the method of the Twyman-Green arrangement, and a method of adjustment which uses the ideal mounting.

If each grating is considered separately, it may be seen to have been mounted in a manner which

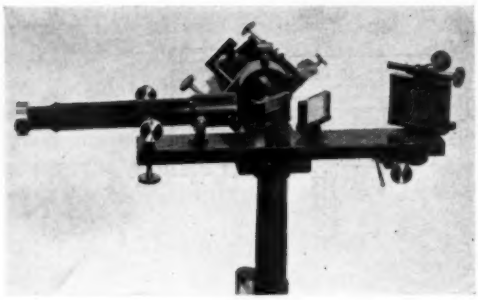

FIG. 2. A grating interferometer. One grating is provided with an ideal mounting, the other with a spectrometer mounting. The grating has been removed from the ideal mounting to show the details of construction.

differs from that known as the Littrow method only by the substitution of the half-silvered plate for a right-angled prism. The gratings may therefore be adjusted independently by following the procedure customarily used with the Littrow mounting. The possibility of employing another procedure in the adjustment appears when a different point of view is taken. The instrument as a whole may be regarded as a Twyman-Green type of interferometer, and may be adjusted accordingly. A particularly satisfactory adjustment may be obtained, however, by employing the facilities of the ideal mounting. Each grating is rotated in any way whatever that will bring the desired line into the field of view. In making a rough adjustment, it is preferable that the slit images from the two gratings should not be superposed. The procedure consists in rotating one of the gratings about its fixed axis; that is, about the axis parallel to the arm of the interferometer. It is possible that, as the grating is rotated in this way, the position of the diffracted beam will shift. The direction of the shift may be observed by noting the position of the diffracted image of the slit with reference to the cross hairs of the telescope, or with reference to the other slit image. If this occurs, any given point in the slit image will describe a circle as the grating is completely rotated about the fixed axis. Rotation of the grating about one of the other axes alters the radius of the circle through which the point moves. Proper manipulation of the slow motion screws reduces the radius of the circle to zero. When the grating has been accurately adjusted to the position of minimum deviation, no motion of the slit image occurs as the grating is rotated about the fixed axis if the line employed in monochromatic. The slight motion which occurs when there is a wave-length distribution across the line is discussed later in the paper. Because of this motion, the adjustment can be made more satisfactorily when spectra of higher orders than the zero order are employed.

From what has been said, it is apparent that the grating in the other arm of the interferometer may be adjusted likewise, if it also is provided with an ideal mounting. When this is done, the two slit images are superposed only if the half-silvered plate accurately bisects the angle between the two beams incident upon the gratings. The first step in the adjustment of an interferometer such as that shown in Fig. 2, which has only one ideal mounting, consists in adjusting the half-silvered plate to bisect the angle between the beams incident upon the gratings. The grating with the spectrometer mounting is then adjusted according to the procedure customary in spectrometer practice. In the procedure, the collimator slit is made parallel to the rulings of this grating. The grating with the ideal mounting is then adjusted as described above. The slit image from this grating is made parallel to that from the grating on the spectrometer mounting. The rulings of the gratings are now parallel to each other and to the collimator slit. The two slit images are not necessarily superposed, however. A lateral adjustment of one of the slit images will probably be required. This adjustment should be made by rotating the grating on the spectrometer mounting about the turntable axis.

III.

Study of the grating interferometer which has been described makes it apparent that interference systems satisfactory for the purposes of many measurements may be obtained whenever single source images of the same wave-length are superposed by any means whatever. It is not necessary, however, to employ for both plates —or for either of them—any one of the procedures which have been suggested for exact adjustment. A very simple mounting of the gratings is effective, therefore, unless sharp definition of the spectral lines is desired. On an

interferometer with such simplified mountings, each arm should be fitted with a turntable for gross adjustments of the gratings. These turntables carry the mechanical devices which constitute the rest of the mounting. The mountings may provide the gratings with freedom for motion about axes of diffraction, but this was found to be unnecessary. It is sufficient to fix the gratings in position with their rulings as nearly parallel to each other and to the turntable axes as can be done without the advantage of optical adjustments. The mounting on one turntable should provide the grating with a fine adjustment about the turntable axis. A slow motion screw built into the other mounting should provide a fine adjustment about the axis of reflection. Thus, in this form of the grating interferometer, the fixed axis of the ideal mounting—which was accessory to the adjustment for minimum deviation and requisite for the best definition obtainable—is dispensed with. In like degree and manner, the axis of diffraction is unnecessary.

Properties of the Grating Interferometer

Observations indicate that the grating interferometer has certain surprising and interesting properties. Considering the fine quality of the optical surfaces which is demanded of the Michelson reflecting mirrors, one might think that the selection of gratings suitable for producing interfering wave fronts would be an exacting matter. Almost any grating seems to be usable, however, and Wallace replica plane reflection gratings have been found to be altogether satisfactory. It is not necessary that the exposed surface of one grating equal that of the other. The total numbers of rulings on the two gratings may differ. They need not be alike as regards grating space. Perhaps the chief point of interest in the grating interferometer is that the spectrum order need not be zero. For this reason, the interferometer is auto-monochromatic. An interesting property also is the production of interference between dissimilar orders, as when the third order from one grating is made to interfere with the second from the other.

The condition most nearly resembling that employed by the Michelson interferometer is realized when the axes of the turntables are parallel and the gratings are rotated in opposite directions to positions of minimum deviation. In order to have a convenient nomenclature, the following definitions will be employed:

1. Positive orders are obtained by rotating the grating in a counterclockwise direction.

2. Any orders, $m_1=k_1$ and $m_2=-k_2$, are complementary if both k_1 and k_2 are positive numbers, or if both are negative numbers.

3. Any orders, $m_1=k_1$ and $m_2=k_2$, are chromatic orders if both k_1 and k_2 are positive numbers, or if both are negative numbers.

When the gratings are adjusted for complementary orders, the virtual sources of the interfering beams are practically parallel, and the fringe patterns are not different from those of the zero order.

When the grating is adjusted for the minimum deviation of some particular λ, the grating is approximately, but not exactly, in the position of minimum deviation for any $\lambda+\Delta\lambda$. It is possible to see what happens by examining the formula for maxima:

$$\sin i=m(\lambda+\Delta\lambda)/\sigma-\sin i'. \qquad (7)$$

Since the grating is illuminated with parallel light, $\sin i$ is constant for all values of $\lambda+\Delta\lambda$. Consequently, the angle of diffraction for $(\lambda+\Delta\lambda)$ will be greater than the angle of minimum deviation for λ. When the gratings are adjusted for the minimum deviation of λ, the beams of wave-length λ from the two arms of the interferometer are superposed. And furthermore, in complementary orders, the beams of wave-length $\lambda'=\lambda+\Delta\lambda$ are superposed if the distances of the gratings from the half-silvered mirror are equal; but in chromatic orders the slit images of wave-length λ' will lie on both sides of the superposed images of wave-length λ. These situations are indicated by the diagrams of Fig. 3 and Fig. 4. It follows likewise for an unresolved distribution $\Delta\lambda$ that in complementary orders the slit images must be exactly superposed to produce observable fringes, but that in chromatic orders fringes will be produced when the slit images overlap without exactly coinciding.

The rotation of the grating through π radians about the fixed axis of the ideal mounting changes the superposition from complementary orders to chromatic orders. Consequently, in the adjustment of the interferometer, it is not possible to remove entirely an apparent relative motion

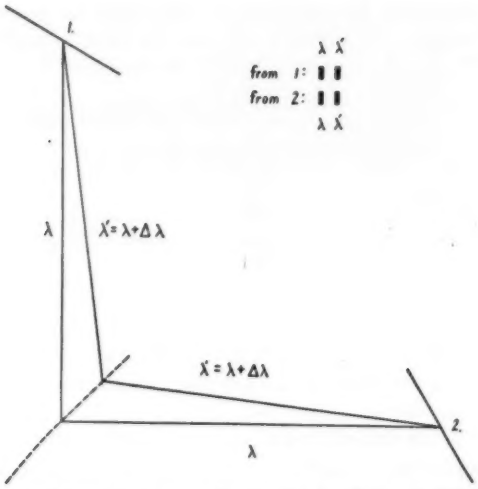

FIG. 3. Scheme for superposing two beams of wave-length λ with gratings set for complementary orders.

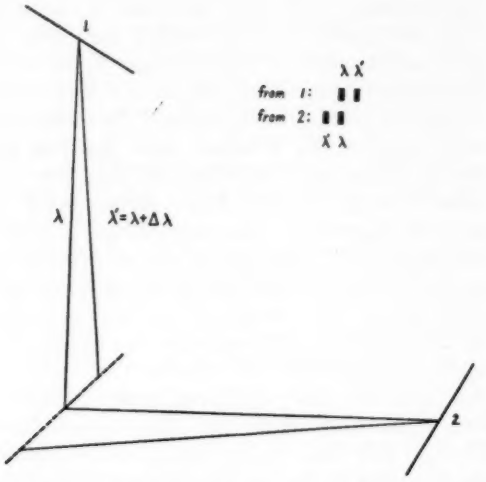

FIG. 4. Scheme for superposing two beams of wave-length λ with gratings set for chromatic orders.

between the two slit images unless the spectral lines are comparatively pure. The nicety of the adjustment is most apparent in complementary orders; if it is desirable to employ chromatic orders, the grating may be rotated about the fixed axis after the adjustment has been made.

When the instrument is fitted with a telescope and a collimator adjusted for parallel light, it is an interferometer of the Twyman-Green type. Either localized bands or infinity circles may be established in the field of view. The collimator and telescope are merely auxiliary, however, in the establishment of any system of fringes. Interferential systems may be observed after the removal of either collimator or telescope, or of both. But the fringe systems in the two cases are not to be confused. Michelson fringes employ the Fresnel type of diffraction; Twyman-Green fringes employ the Fraunhofer type of diffraction. That this distinction applies to interference systems of the zero order can be seen from an experiment described by Williams.[2]

CONCLUSION

Merely the properties of the grating interferometer are worthy of attention. But the instrument has significance aside from the fact that it is auto-monochromatic, that it produces interference between higher orders and different orders, etc. The principle upon which it operates is that of the spectrum order of the interfering beams may have any value. The Michelson arrangement employs this same principle, but in a specialized case. The general idea is almost, but not quite evident in the action of the Twyman-Green interferometers. It is boldly used, but not explicitly stated by Williams[3] in testing the reflection echelon. Thus the grating interferometer is significant because it affords a more comprehensive and accurate understanding of the exact action of familiar forms of interferometers. Moreover, it illustrates the close relation between the laws of reflection and the laws of diffraction; and it suggests the possibility of altering the designs of other optical instruments by the substitution of gratings for mirrors.

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[2] W. E. Williams, *Applications of Interferometry*, E. P. Dutton and Company (1930), p. 71.

[3] W. E. Williams, *Studies in Interferometry—II*, Proc. Phys. Soc. **45**, Part 5, 699 (1933).

Ninth International Congress of Scientific and Applied Photography, Paris, July 7–13, 1935

THE first International Congress of Photography was held in Paris in 1889. Since that time meetings have been held at intervals of three to five years, excluding the four-year war period, in Liège, Brussels, London and Dresden, in addition to Paris.

The next Congress, the Ninth, will be held again in Paris next year, 1935, from Sunday, July 7, to Saturday, July 13. The meetings will be held in the rooms of the Société Française de Photographie et de Cinématographie, 51 rue de Clichy, Paris (IX^e).

The Ninth Congress will be organized on lines similar to those of previous congresses. The active organization will be in the hands of a French Committee consisting of representatives of many of the scientific, photographic and allied societies of France, and headed by the French Photographic Society. The arrangements in other countries are made by the local National Committees. These committees have been established in many countries to deal with the proposals and recommendations of the congresses, to present material to the congresses for international consideration, to arrange for a series of first-class papers on appropriate photographic subjects to be submitted to the congresses, and so on.

The Secretary of the American Committee is Dr. Walter Clark, Research Laboratories, Eastman Kodak Company, Rochester, New York. There are two sub-committees in this country, dealing, respectively, with Sensitometric Standardization and Motion Picture Standards.

Sections: The Ninth Congress will be divided into several Sections, concerned with:

1a. Latent image.
1b. Sensitive materials and their manipulation.
1c. Sensitometry and photographic photometry.
2. Cinematography; general considerations, sound on film, standardization.
3. Scientific and technical applications of photography and cinematography.
4. History of photography, documentation and bibliography; photographic instruction; pictorial photography.

Among the subjects about which there will undoubtedly be lively discussion will be the standardization of methods for the determination of speeds of negative materials, and the standardization of the dimensions of sub-standard motion picture film for sound-on-film purposes.

Membership: The Congress will include the following classes of Member: Founder Members (minimum subscription 1000 French francs); Subscribing Members (minimum 300f); Active Members (125f); Associate Members (25f).

With the exception of the Associate Members, all the members attending the Congress will receive, at the Congress Secretary's office, the texts of all Communications and Reports, as well as résumés of these texts in various languages. They will eventually receive the volume of Proceedings.

Papers: Reports and papers submitted to the Congress from the United States should be sent in duplicate to the Secretary of the American National Committee by February 1, 1935, so that he can have them in the hands of the General Secretary by March 1, 1935. Only in these circumstances can they be translated and distributed.

Reports and résumés can only be given to Members attending the Congress if their subscriptions have reached the Société Française de Photographie et de Cinématographie by June 1, 1935.

Languages: Discussions will be held in one of the following languages: English, French, German. Authors should write their texts where possible in one of these languages.

Entertainments: Entertainments, visits to factories, and scientific institutions, excursions and a banquet will be organized, and all Members of the Congress will be able to attend. For Associate Members, a Committee of Ladies will organize tours in Paris and the surroundings, and visits to buildings of interest, during the sessions of the Congress.

Information: For further information about the Congress, application should be made to the Secretary of the American National Committee, Dr. Walter Clark, Research Laboratory, Eastman Kodak Company, Rochester, New York.

W. CLARK

Eastman Kodak Company,
Rochester, New York,
July 9, 1934.